DOCUMENT RESUME

ED 065 339 SE 014 404

AUTHOR Lockhart, John
TITLE Guide for Volunteers in Mathematics.
INSTITUTION Oklahoma City Public School System, Okla.
PUB DATE 71
NOTE 73p.

EDRS PRICE MF-$0.65 HC-$3.29
DESCRIPTORS *Activities; *Arithmetic; *Elementary School Mathematics; *Instruction; Instructional Aids; *Instructional Materials; Learning Activities; Paraprofessional School Personnel; Tutoring; Volunteers

ABSTRACT

This document is intended as a guideline for volunteers who work with elementary children in mathematics in the Oklahoma City School System. Following referral and progress sheets designed to increase teacher-volunteer communication about the student, the bulk of the book is a list of involvement activities to assist volunteers in helping students in mathematics. These activities cover elementary topics that have been identified as problem areas: number concepts, place value, the four operations with whole numbers, fractions and their equivalents, and the four operations with fractions. The activities generally allow the student to "do" something, and materials are inexpensive and easily prepared. (JM)

ED 065339

U.S. DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
OFFICE OF EDUCATION
THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL OFFICE OF EDUCATION POSITION OR POLICY.

VOLUNTEER • CHILD • TEACHER

GUIDE FOR VOLUNTEERS IN MATHEMATICS

HELPING HANDS

OKLAHOMA CITY PUBLIC SCHOOLS

Training of Professional Personnel in Effective Utilization of School Volunteers and Training of Student Tutors

Developed in cooperation with the Bureau of Educational Personnel Development, U. S. Office of Education.

EPDA 1971

GUIDE FOR THE VOLUNTEER

IN

MATHEMATICS

Oklahoma City Public Schools
1971

OKLAHOMA CITY PUBLIC SCHOOLS

Dr. Bill Lillard, Superintendent
Dr. Mervel Lunn, Assistant Superintendent,
Instructional Services
Dr. Otis Lawrence, Director of Curriculum
Mrs. Lois Rogers, Volunteer Program Coordinator
Mrs. Nelda Tebow, Project Training Specialist

COMMITTEE

Mr. John Lockhart, Chairman, Consultant
Mrs. Allene Mize, Consultant
Dr. William J. Coffia, Oklahoma City University
Consultant
Mrs. Mary Hepp, Secondary Teacher
Mrs. Olivia Nixon, Secondary Teacher
Mrs. Gerri Wade, Volunteer

TABLE OF CONTENTS

To the Volunteer. 1
Goal of the Volunteer Program 2
Role of the Volunteer 2
Referral Sheet. 3
Progress Sheet. 5
Involvement Activities. 7
Number Concepts. 8
Place Value, Reading, and Writing Numerals 10
Addition Facts 15
Subtraction Facts. 22
Multiplication Facts 25
Division Facts 28
Operations - Addition. 33
Operations - Subtraction 39
Operations - Multiplication. 41
Operations - Division. 44
Knowledge and Meaning of Fractions 48
Equivalent Fractions 52
Addition and Subtraction of Fractions. 59
Multiplication of Fractions. 61
Division of Fractions. 64
Bibliography. 66
Glossary. 67

To The Volunteer

You are to be an associate to the teacher in helping each child develop his full potential -- a challenging but most rewarding assignment. Your particular role will be to help provide the student with whatever additional practice and individual attention areneeded for him to master the desired concepts.

While your confidence may be bolstered by this manual, it is not intended to limit your own resourcefulness.

This manual has been arranged by levels as a matter of convenience in locating materials appropriate to the level of accomplishment of the student. Most activities concerned with a particular skill, however, may be adapted to the needs of a student at any level. In order to acquire pertinent vocabulary and an understanding of the mathematical concepts and teaching techniques being taught and used, you may wish to review the teacher's edition of the mathematics textbook.

You are a welcome and essential addition to the educational endeavor in our city, and the Oklahoma City School System encourages you to make use of all available personnel and services which it provides.

Goal of the Volunteer Program As It Relates to Mathematics

To provide the student an opportunity to acquire a working mastery of basic mathematical skills and concepts by helping the teacher to personalize instruction.

Role of the Volunteer

1. Establish a supportive interpersonal relationship with the teacher.
2. Develop with the teacher some effective method of communication concerning the student's needs and progress as well as the activities in which he is engaged.
3. Establish a supportive interpersonal relationship between yourself and the student.
4. Help each student build a more positive self-concept. (The negative self-concept is inhibitive and blocks potential growth.)
5. Provide the student with opportunities for successful and enjoyable learning experiences.
6. Become involved with the student(s) in the learning process.

REFERRAL SHEET

The following form is one way by which the teacher can communicate the difficulties of the student to the volunteer. The teacher should diagnose the weaknesses of the pupil and make suggestions and comments which may be helpful to the volunteer.

REFERRAL SHEET

Student's Name _____ Teacher's Name _____

Areas of Difficulty	Teacher's Suggestions or Comments
I. Number Concepts a. Comparison of sets b. Number of a set c. Ordering numbers	
II. Place Value	
III. Number Facts a. Addition b. Subtraction c. Multiplication d. Division	
IV. Operations with Whole Numbers a. Addition b. Subtraction c. Multiplication d. Division	
V. Fractions a. Meaning of fractions b. Equivalent fractions c. Operations with fractions 1. Addition 2. Subtraction 3. Multiplication 4. Division	

A PROGRESS SHEET

The following form is a progress sheet which may be used to aid the volunteer and the student in keeping a record of student's progress. Often a listing of our accomplishments is a good thing to have. (Especially on days in which we do not live up to our expectations.)

This progress sheet may be useful in teacher-volunteer conferences. You should decide about its usefulness for you. Perhaps this form will not work for you in your situation. You should be the judge. The heading "Suggestions for Our Next Session" can be used to set goals for the future, to discuss difficulties, and to plan new projects which might help.

Our "Progress" might be as simple as discovering something we cannot do at this stage but will permit us to work out a plan for overcoming the difficulty.

PROGRESS SHEET

Name _____

Date	Time Spent	What We Did Today	Progress	Suggestions For Next Session

INTRODUCTION

INVOLVEMENT ACTIVITIES

This instructional package was developed to assist volunteers in helping students in mathematics.

Certain areas in which many students experience difficulty have been identified. Activities have been selected, described, and illustrated for the volunteer's use in helping students overcome these difficulties. Some effort has been made in arranging the activities in ascending order of difficulty for convenience in locating those experiences which match the needs of the pupils. It is assumed that the teacher will diagnose the needs of pupils and will then communicate those needs to the volunteer. These communication lines should remain open between teacher and volunteer so that learning styles, instructional materials, and techniques can be matched.

The activities are designated as Involvement Activities because it is felt that learning is more likely to occur if the learner is a "doer." Furthermore, it is felt that a volunteer, being free of other responsibilities, can give the pupil added opportunities to become involved as a "doer." (Consult activities listed in the manual.)

The materials recommended are inexpensive and easily prepared. Research indicates that manipulative materials are effective, but their use by the teacher will determine their ultimate value. Since the volunteer is in the teacher role, he should try to ascertain the specific errors which a pupil is making and give appropriate help. The enthusiasm of the volunteer and praise of pupils' efforts are regarded as essential.

VOLUNTEER - MATHEMATICS

NUMBER CONCEPTS

1. Make up sets for and with children that can be compared easily by observation (real things or flannel board representations).

2. Have pupils consider the set made up of all boys in the room. Ask some boys and some girls, "Are you a member of this set?" "Why or why not?" Continue with other sets of this kind.

3. Have children draw pictures of specific sets, such as their family, toys, things that grow in the yard, etc.

4. Display several sets; describe one, and ask a pupil to draw a ring around it.

5. Clown Game -- Draw or cut out a picture of a clown holding a group of balloon strings. Use flannel on the back of the picture if a flannel board is available; if not, tape it on the chalk board or draw it with colored chalk. Cut out several sets of balloons of various colors and shapes, some round, some shaped like animals, etc. One set could be equivalent to the set of strings, one could have one more balloon than the set of strings, and one could have one less. After you have provided some direction through your questions, let small groups of children experiment freely with these materials.

6. From pictures have the child or children pick out specific sets, and then, by matching, determine which is larger.

7. Give each child opportunities to match a given set by making equivalent sets, such as: a set of children matched with a set of books.

8. Give each child an individual chalk board or 9" x 12" sheet of paper, write the numeral one on the board. Ask pupils to draw a set that matches the numeral. Continue writing numerals through five, having children draw sets to match the numerals.

9 Cut out of cardboard or paper five "stepping stones," and number them. Draw an imaginary river on the floor with chalk. Arrange the "stones" in the "river" and have the child cross the "river" in the order of the numbers.

10. Set up three sets, each with one more than the set before it. Have children make the next set with one more up to five sets. Help children to see the stair-step arrangement that occurs.

11. Show a set, and ask children to draw or show a set with fewer members and one with more members. (Sponge blocks may be used.)

12. Place twenty or more felt objects on flannel board. Pass pieces of yarn out to three children. Ask them to ring any set of six. Remove yarn and continue by asking different children to ring sets of seven, then eight and finally, sets of nine.

13. Have a child write his or her name on the board. Ask the children if the name is of that child. Lead them through a discussion to see that numerals name numbers.

14. Arrange sets of 1-5 in any order. Guide the child to determine which one is smallest; put it at the top and then determine the largest. Guide the class to put the others in order; then attach numerals by counting each set.

15. Play a game by putting easily identified objects in three boxes and nothing in another. With eyes closed, ask a child to describe each set by feeling of its members.

16. Start with a set of objects, remove one at a time, ask the number that is left until there is none left. Then what number describes how many are left?

17. Play a game using cards which have numerals on them. Hold one card up and tell the children that the numeral you see tells you the number of times you are to do the things I tell you to do, or the number of things you are to bring to me. For example:

    1. Clap this many times.
    2. Bring me this many crayons.
    3. Point to this many chairs.
    4. Draw this many balloons, etc.

18. Have an object hunt. Then allow two or three students working together to create different sets with the objects and then describe the set.

19. After the child has worked with sets of objects and pictures, begin to use sets he or she cannot see, such as sounds around them or characters in an imaginary story

20. Mark a point on the floor. Ask a small child to come to the point and stand. Ask how many steps she has taken. "None." Ask what numeral tells how many steps. "Zero." Have a child put the zero numeral card beside his or her feet. Ask the student to take a step. Now what numeral describes the number of steps? "One." Again have a child place the numeral. Continue through ten. Ask if she or he could go on taking steps to help them see a number line goes on and on. Have a larger child do the same thing in another spot. Compare the two leading the class to see that the larger child's steps are farther apart, but in each case the steps each took are about the same size. Transfer to the board by imagining steps were taken on the board. Have a child indicate a starting point and ten steps. Put a dot at each point, and number each point to help establish a picture of a number line.

21. A number line for each child is very helpful. Give children practice in counting the spaces and locating the numerals. Have children read numerals from left to right and lead them to discover that as we go to the right the numbers named are greater. Start at seven and point to each numeral as you go left along the number line and lead children to discover that the numbers named are less.

VOLUNTEER - MATHEMATICS

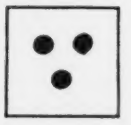
PLACE VALUE - READING AND WRITING NUMERALS

1. Give children many experiences grouping by tens using all kinds of materials. Partition sets into sets of tens and ones. Record the number of tens and ones. Use strings of beads, spools on a board, piles of blocks or sticks, and put rubber bands around the sets of tens.

2. Be sure through several counting experiences that children have a sound idea of the sequence of numbers through ten and twenty. Then give them experiences grouping sets by tens.

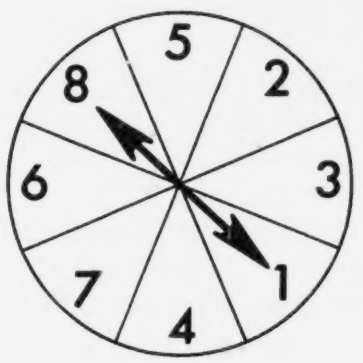
3. Have children show 10 ones on their hands by 10 outstretched fingers; then have them change 10 ones to 1 ten by clasping their two hands together. Help them see that 10 ones and 1 ten are two ways of talking about the same number.

4. Use pattern cards, numeral cards, and number name cards; distribute these among the children. Name a number and ask all who have that number to come to the front.

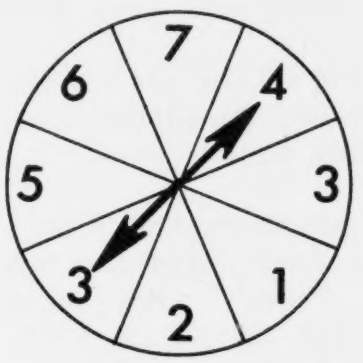


5. Use different pictures of several tens and have the children describe them in different ways, such as "Which picture shows 3 tens?", "Which shows 2 tens?", etc.

6. Tens and ones - use two number wheels as shown:



TENS



ONES

Have the student spin the pointers and add the ones - then the tens. In the example shown, the answer would be 97. He may either write or tell the number. See packet for spinners.

7. Every firsthand need for reading numerals should be utilized as a learning situation. Page numerals, room numerals, calendar numerals, clock numerals, and many other numerals should be brought to the child's attention.

8. Have children count sounds as you tap the desk or a triangle. After a set of 10 taps, child holds up a finger. Stop on every multiple of 10, and have children tell you how many sets of ten taps there were.

9. Distribute a set of flash cards to the pupil or small group, bearing a number from 1 to 100. Begin counting aloud slowly. The children hold up their cards as the numbers are called.

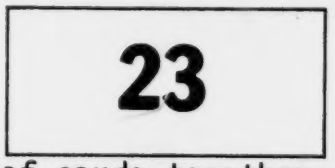
10. Prepare a set of 100 cards showing the numbers 1 through 100. Select any 10 cards which are in sequence (for example, the cards for 20 through 29 or for 45 through 54, etc.). Line the cards up in order on the chalk tray and ask a pupil to leave the room. Move one of the cards so it is out of order. Ask the child to return and put it back in order. If he does it correctly, he may choose someone to leave the room while he changes a card.

11. Use the 100 chart and have all pupils play the game "What's the Numeral?" Have the children close their eyes. After you have covered a numeral, have them open their eyes and tell what numeral is covered. Repeat, using other numerals.

12. Set up work problems involving coins, such as "You have 3 dimes and 4 pennies. If your mother gives you 2 dimes and 3 pennies, how many coins do you have in all?" Illustrate the problem by drawing coins on the board or by using real or play money. Point out that each dime has the same value as ten pennies. Have pupils join the sets of dimes and the sets of pennies and determine the number of the new sets and the numeral which tells how many sets in all.

13. Use sets which when grouped by tens have some left over and discuss how to write the numeral, using expanded notation, and then the numeral.

14. Game - OLD HAT
Write a 1 or 2 number on each of 12 to 24 cards. Then prepare a corresponding set which shows in bundle numbers the equivalent of each number card as shown.



Mix both sets of cards together, withdraw one card (leaving a card that has no matching card), and deal all cards to 2 to 6 players. Each player should match cards in his hand to make books of two cards each. Players should then take turns drawing cards from one another in an attempt to complete books which can be discarded. The pupil who holds the odd card at the end of the game is the "Old Hat."

15. L-E-G. Cut ten index cards in fourths, making a total of forty small cards. The cards are labeled on the front and back as shown below, the letters on one side and the numerals on the other. Each student should have his own deck of cards.

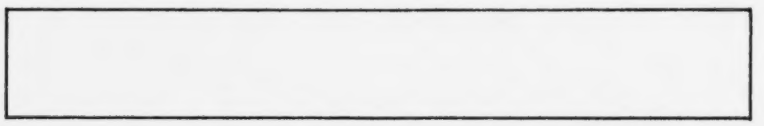
6 L	78 L	682 G	541 L
7 L	41 L	794 G	569 G
24 L	104 L	547 G	983 G
115 L	140 L	571 G	741 G
126 L	68 L	617 G	641 G
178 L	984 G	981 G	394 L
42 L	381 L	683 G	542 E
943 G	362 L	372 L	78
68 L	174 L	84 L	848 G
978 G	789 G	79 L	681 G

VOLUNTEER - MATHEMATICS

Instruct the student as follows: you will find a set of cards spread out on the table. Make sure the numerals are facing up. On the reverse side of each card is written one of the following:

L (for "less than")
G (for "greater than")
E (for "equal to")

The object of this experience is to turn over as few cards as possible before you come to the card with E on the reverse side. Turn over as many cards as you need to, one at a time, making a tally mark in the space below each time you turn over a card, After you have found the E card, add up the tally marks and give the total.



Number of Marks _____

16. SECRET CODE

Make a deck consisting of fourteen index cards marked on front and back as shown below:

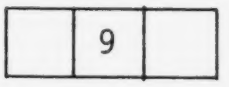
Front	Back	Front	Back
35	T	801	R
72	H	810	I
173	I	811	G
617	S	907	H
671	/	970	T
761	I	978	
796	S		
900	/		

Mix the cards thoroughly. Have the student arrange them in order of their value from least to greatest and then turn them over to discover a secret message.

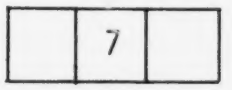
17. You will need three cubes with numerals on them for this activity. Each cube is different. The numerals on them are shown below:

1, 2, 3, 4, 5, 6
3, 5, 6, 7, 8, 9
1, 2, 3, 7, 8, 9

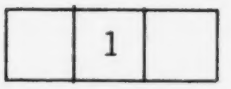
1. What is the greatest number you can make by lining up the cubes next to each other?



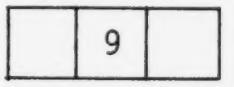
2. What is the greatest number with 7 in the tens place?



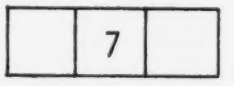
3. What is the greatest number with 1 in the tens place?



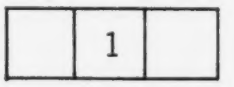
4. What is the smallest number with 9 in the tens place?



5. What is the smallest number with 7 in the tens place?

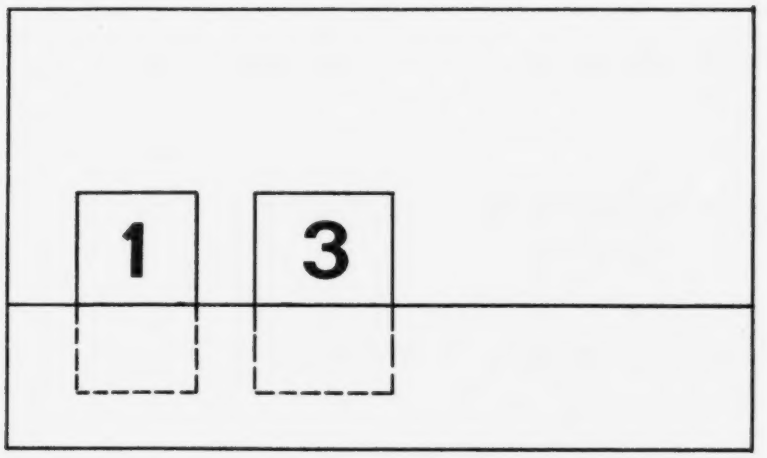


6. What is the smallest number with 1 in the tens place?



VOLUNTEER - MATHEMATICS

EVERYBODY SHOW



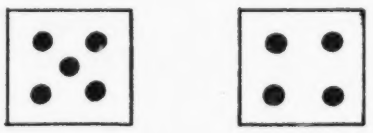
Construct a pocket chart for each individual in the group to be instructed. Also, prepare cards on which appear the numerals 0 through 9 (or 19, depending on the use). The one shown could represent place value (1 ten and 3 ones) or it might represent the sum of 7 + 6. The volunteer issues the challenge which will help the child attain his skill goal and the pupil(s) respond by placing appropriate cards in the pocket, and then showing that response to the instructor.

ADDITION FACTS

1. Sets of many different kinds can be used to develop these ideas. Demonstrate a set and have the children count and tell the number. Demonstrate a second set and have the children count and tell the number. Join the two sets and determine the number of the larger set. Repeat, using numbers orally to tell what was done. (Pictures cut from magazines and pasted on cards can be used as sets.)

2. Let children use discs or some other manipulative objects to join and separate sets as you do them at the flannel board. Start with a set, join a set, and discuss how many in the larger set. Start with the same set and separate from the same set which was joined. Each time record with numerals what was done.

3. Use sets or perception cards and allow the children to discover that in joining sets or adding numbers the order does not change the final result. Let the class give other instances where order makes no difference, such as putting on a coat and hat. Example:



4. Demonstrate joining two sets and ask a child to record, using numerals, what happened.

5. Choose two boys to come to the front of the room: ask a girl to stand in front of them. Suggested questions: How many boys are there? How many girls? How many children in all? Have the girl stand behind the two boys. Ask: How many girls? How many boys? How many children in all? When the children changed places, did it change the number of children in all?

6. Place 7 plastic bottles or other objects before the class and ask the children to tell how many bottles they see. Next ask eight children to stand and have the class count the number of children standing. Tell those children standing to pick up 1 bottle each. Ask again, "How many bottles are there?" How many children? How many more bottles do we need to make eight? You can see that eight is one more than seven.

7. A beanstalk may be shown by drawing a line from the top of the chalkboard to the bottom. Draw four large leaves on one side of the stalk and ask the children to tell how many leaves they see. Draw four more leaves on the other side of the stalk and ask the children to tell how many more leaves they see and how many leaves there are all together. From this demonstration develop these equations:

$4 + 4 = \square$ $\qquad$ $4 + \square = 8$ $\qquad$ $\square + 4 = 8$

8. Children could make own addition flash cards with all the combinations through ten on one side and the answers on the other. This could be used with two children, one asking the other, or in small groups with the teacher.

9. Use exercises such as the following to help develop skill in working equations with parentheses. Let the children use sets and partition them if they need to.

   a. $4 + (2 + 3) = \square$
   b. $4 + (\square + 1) = 10$
   c. $\square + (2 + 3) = 9$
   d. $6 + (3 + \square) = 10$

10. Using sets, help the children to see that it makes no difference which way the three sets are joined; the results will be the same.

11. "Give Me Another Name" - Use numeral cards to show a numeral and say, "Give me another name." If you flash six, the child might say 5 + 1, or 4 + 2, if he is reviewing addition. If he can't give another name, offer a set of counters and let the child figure it out. If there is more than one child, a variation might be to let a child who gives a correct answer give the number and call on someone to "give another name." This could be used with all operations and to check understanding of place value.

12. Addend - Sum Game - A questioner asks, "I'm thinking of a sum of 8 and an addend of 5. What is my other addend?" or "I'm thinking of an addend of 2 and an addend of 6, what is my sum?" The student who answers correctly becomes the next questioner.

13. Racing Car Game - Each player places a marker (his car) at the starting point (use a number line on the floor or board). A player rolls dice; either adds, subtracts, or multiplies the two numbers to determine the number of spaces he or she moves. Determine a finish line and whoever reaches it first wins. A set of cards may also be used with instructions, "miss a turn," "go back to start," etc., to vary the game for older children.

14. What's My Rule?- Ask the student to name a number less than ten. You name a number two greater. Repeat the process until the student can give you the rule for naming the second number.

15. Domino Bingo - Draw nine rectangles the approximate size of dominoes on each of three cards and insert numbers according to the scheme pictured on the next page. Then turn a set of dominoes face down and have three players take turns drawing dominoes. If the sum of the domino pattern is equal to a number on the pupil's card, he covers it with the domino; otherwise, he places it in a discard pile which is to be mixed and reused if nobody wins by covering all squares before the original pile is depleted.

Example for Number 15.

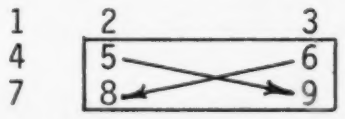
1	6	10
5	3	9
7	8	4

3	5	4
9	7	11
6	8	2

2	4	6
5	6	7
10	8	12

16. Bingo - Instead of using regular discs provided, shuffle and use flash cards to call. The student must work the combination before he places a marker on his card. Keep the cards separate so as to check when someone bingos. It is also possible to make up specific combinations that need drill.

17. To provide drill in addition, and to stimulate some creative thinking, have the children set up numbers in a table of 3's like this:



Direct them to frame any four numerals as shown and add the numbers diagonally. Ask: "What did you find?" Suggest they set up a table by fours and see what happens, or one by fives.

18. Give the children a piece of paper with a pre-drawn line on it and let them construct a number line and use it to solve number sentences and discover number facts. Let them choose their own unit for the distance between points.

19. It is useful to construct a number line on the floor to illustrate problems.

20. Simple magic squares can be used to create some drill in adding three numbers. Put exercises like the following on the board:

Find the sum for the first row.
Find the missing numbers (those in parentheses) so all the sums will be the same up and down and across.

2	3	4	= (9)
4	1	(4)	= (9)
(3)	(5)	(1)	= (9)

Some children may be able to make their own squares if given just the sum.

21. Put this activity on the board and let the children circle or mark in some way the ones that are true sentences:

4 + 5 = 5 + 4	6 + 1 = 2 + 6
3 + 2 = 2 + 3	3 + 4 = 4 + 3

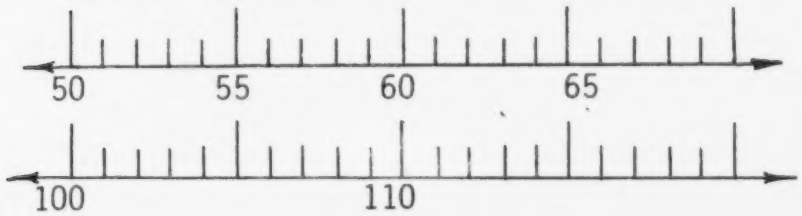
If they have difficulty, let them use sets to decide which ones are true.

Using three sets instead of two, work through an activity similar to number one.

22. Use the variation of Bingo called "Quizmo" for addition and subraction combination practice. (Note: game is a commercial product and can be purchased in all book stores. Also available in multiplication and division combinations.)

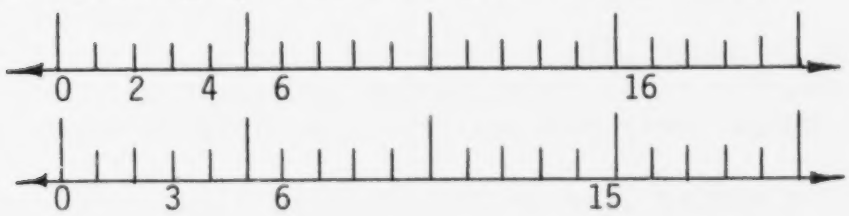
23. The number line can be used as an effective teaching tool for many skills such as:

a. Number sequence.



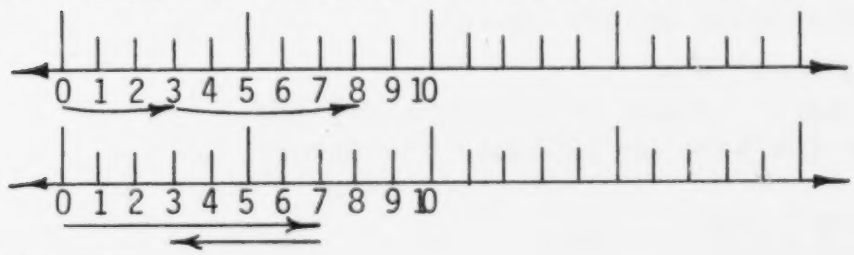
Label the points in between.

b. Counting by 2's, 5's, 10's or any number.



Finish the patterns.

c. Illustrate addition and subtraction facts.



3 + 5 = 8

7 - 4 = 3

24. "ADD-ICE"

Purpose:

1. To teach mental arithmetic
2. Column addition drill

Materials:

1. 3, 4, 5, or 6 dice marked according to the type of drill needed. (Milton-Bradley beads make good dice.)

Example 1 - Each dice could be marked 1 - 6.

Example 2 - Four dice might be marked 0 - 5 and one or two 3 - 9.

Example 3 - The numbers 6, 7, 8, 9 could be randomly placed on the faces of three dice or more.

Procedure: (two or more players)

1. Each player rolls the dice in turn. He must give the sum of the numbers showing on the dice.
2. A point can be won only in case of an error. If the sum named by the player is incorrect, the first person to indicate "error" and to give the correct sum scores a point.

25. Bean Bag Toss

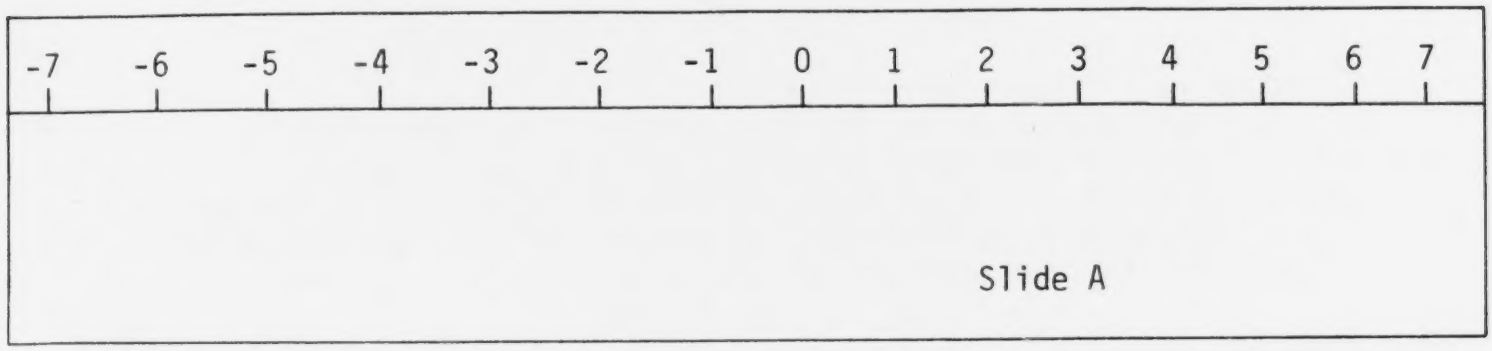
3	7	2	1
8	5	9	0
4	6	3	7

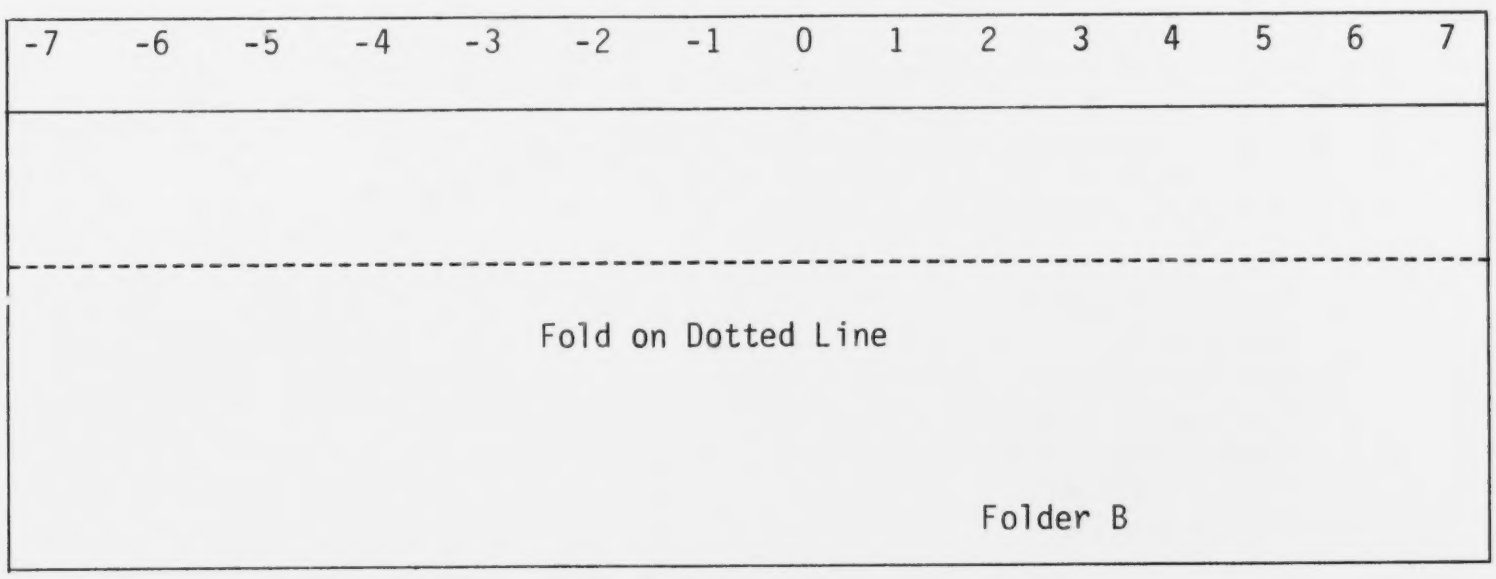
Toss two bags.
Land on 7 and 5?
Add 7 + 5.
Get sum of 12?
You scored a point.
Keep score. Play
with a partner.
Try to win.

VOLUNTEER - MATHEMATICS

INTEGER SLIDE RULE

Purpose: To help the student add and subtract positive and negative integers.





Instructions for making slide rule:

1. Cut out slide A and folder B from stiff paper.
2. Fold back folder B on the dotted line.
3. Place slide A inside folder B. The number line on A will be above the number line on B.

Examples:

A. $-5 + 6 = N$

First, pull slide A until 0 is over the -5 on folder B. Then on slide A find 6. The answer N is directly under the 6.

ARITHMETIC GRID

(Sometimes referred to as addition table.)

	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

This grid can be used for addition, subtraction, multiplication, or division facts. It is placed in the manual so that many copies may be prepared when needed.

SUBTRACTION FACTS

1. Start with a set and have the number of the set determined by the pupil. Separate a smaller set from it and ask how many are in it. Then how many are left in the remainder set? The oral description as well as the manipulation will help prepare the children for subtraction.

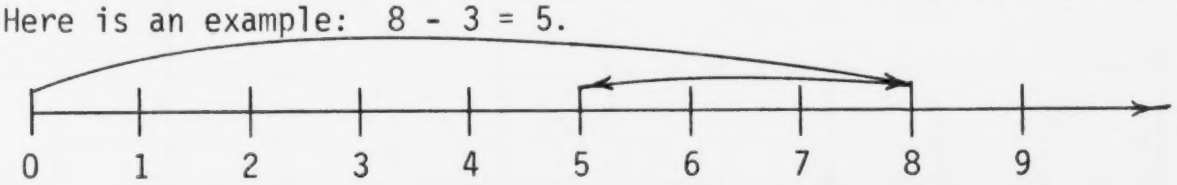
2. On a flannel board, show a set containing, for example, five members and then ask the pupils to give the number of the set. Have the numeral five put in place. Lay a string between a subset with three members and a subset with two members. Have the numerals three and two put in place under these subsets. Ask a pupil to remove the subset with two members and then ask, "What is the number of members left?" Tell the pupils, "We can write a sentence that tells about the numbers of these sets." Show five subtract two is three.

3. Have children make subtraction fact cards (flash cards). These are for individual use and should be mini-cards for slipping in one's pocket. Encourage the student to study his set frequently with or without a partner.

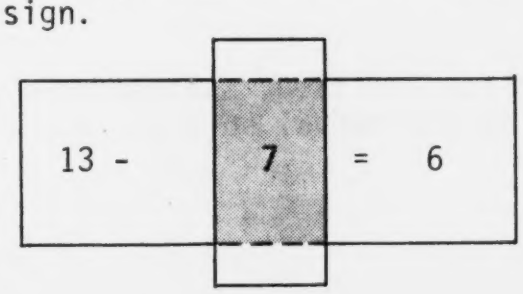
One side
like this.

One side
like this.

4. Use the number line to illustrate the inverse relationship between addition and subtraction. This will aid the student in learning subtraction facts. Here is an example: 8 - 3 = 5.



5. Use the relationship cards for helping children understand the inverse relationship between addition and subtraction. The crosspiece goes completely around the card and slides along so that you can cover either the sum (13 in this example), missing addends (7 and 6 in this example), the equal sign, or the minus sign.



See SHOW ME game, Addition Facts Section, for other ideas.

6. Use the addition table to help children gain knowledge about inverse relationships. A section is shown here in order to help the volunteer work with children using this device:

	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	6
2	2	3	4	5	6	7
3	3	4	5	6	7	8
4	4	5	6	7	8	9
5	5	6	7	8	9	10

Start, for instance, with a sum 7, glance at the top of the columns and the left end of each row in which seven appears. You will find that 4 is one of the addends and 3 is the other. Also that 5 is an addend and 2 is the other. Of course, there are others, 7 - 6 = 1; therefore, 6 is one addend and 1 is the other. This brief paragraph included only for clarification.

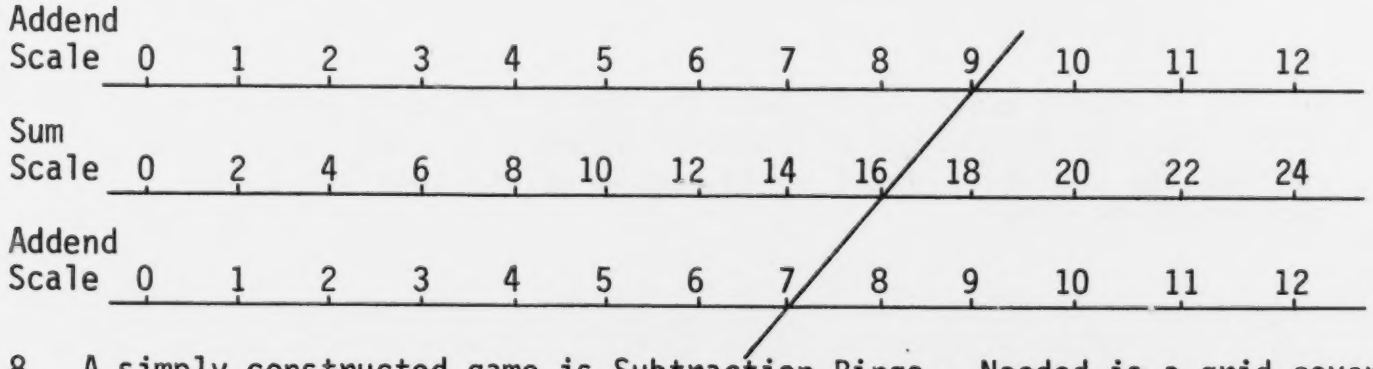
7. The idea that subtraction undoes addition can be illustrated on nomographs. For example, 16 - 9 = ☐ may be thought of in either of these forms:

sum - addend = addend
16 - 9 = ?

or

addend + addend = sum
9 + ? = 16

On the nomograph below, this problem has been solved by placing a straight-edge so that it falls on 9 on one "addend" scale and on 16 on the "sum" scale. The missing addend is found on the other "addend" scale:



8. A simply constructed game is Subtraction Bingo. Needed is a grid cover card and a set of subtraction flash cards. Volunteer shows the flash card and pupil(s) cover the missing addend (difference) when it appears on their cards. Winner calls Bingo when he has row, column, or diagonal completely filled.

9. The game, "Give Me Another Name," found in the Addition Facts Section of the manual is adaptable to subtraction.

VOLUNTEER - MATHEMATICS

MATHEMATICAL SENTENCES - A GAME

To play this game, each of two players has three decks: (1) a deck of "partially complete" mathematical sentences, such as (3 x 13) + (8 x 6); (2) a deck of symbols as $<$, $>$, =, $\neq$ (make several each of these); and (3) a deck of "answers," 76, 39, 101, etc.

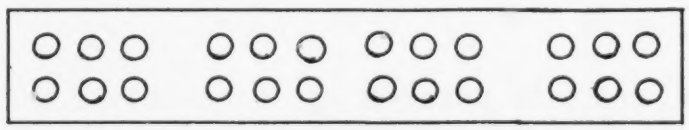
Each player places his decks face down in front of him. They take turns drawing and completing the mathematical sentence. Our example might look like this: (3 x 13) + (8 x 6) $\neq$ 101. This is obviously a true mathematical sentence. The player who made this sentence moves his peg up one notch. Then it is his partner's turn. Suppose he turns up the following cards:

17 - (3 + 7) = 13. This is <u>not</u> a true mathematical sentence. Therefore this player doesn't get to move. The first player to move his peg to the end of the score stick is the winner.

Materials needed:

1. Decks of cards to represent whatever skill need the pupil has. (The game is quite versatile and can be used for learning facts or as an evaluation scheme.)

2. A score stick with two pegs, each a different color. It can be made like this diagram. Use a length of 1' x 2" board and bore holes part way through so that pegs can be inserted.



2 inches in width

10 or 12 inches in length

Hint: On the "symbol" cards, designate the <u>top</u> of the card with a red line. This is so one can distinguish between $<$ and $>$. You can see the confusion which would result unless such a distinction could be made.

MULTIPLICATION FACTS

1. The skill goal with respect to multiplication facts is "quick recall."

2. Lead into multiplication with the idea of equivalent sets of objects, either real things or pictures. For instance, have pupils determine the number in the union of two sets of three as pictured. The pupil may see this set operation as repeated addition.

3. The understanding of the meaning of multiplication may possibly be enhanced by building array patterns. Using counters or objects, have the pupils arrange different sets into array patterns!

```
0 0 0
0 0 0
0 0 0
0 0 0
```

This is called a four by three array. (Four in each column and three in each row. It is shown as 4 x 3.

Have the children bring in materials which show arrays. (Example: egg cartons.)

Have them see if a particular number (6) can be arranged in more than one array:

```
0
0
0
0
0
0
```
6 x 1 array

```
0 0 0 0 0 0
```
1 x 6 array

```
0 0 0
0 0 0
```
2 x 3 array

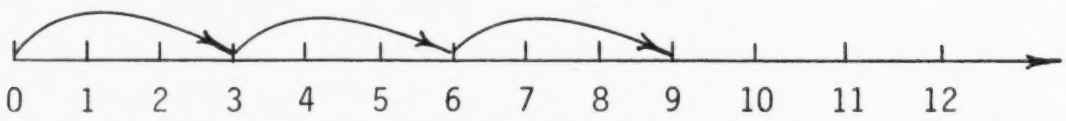
```
0 0
0 0
0 0
```
3 x 2 array

Use arrays to solve the following and note the pattern of the products:

1 x 4 = [4]	4 x 4 = [16]
2 x 4 = [8]	5 x 4 = [20]
3 x 4 = [12]	6 x 4 = [24]

What then would 7 x 4 be or 8 x 4?

4. Use a number line and show moves of three.



Ask how many moves of three from 0 to 9. Have them give the multiplication sentence (3 x 3 = 9). Vary using moves of 2, 4, 5, etc.

5. For number line activities have children draw a number line through 16. Let them illustrate: 2 x 8 and 4 x 4.

6. Construct a set of multiplication cards with either the first or second factor missing. Also some with missing products. Hold up one card and let the child supply the missing factor, or product, and tell which name applies.

$2 \times \square = 0$ $\qquad\qquad$ $5 \times 2 = \square$

7. Write multiplication sentences, such as $6 \times 4 = \square$ on blue cards and $4 \times 6 = \square$ on orange cards. Lay one set (the blue) face-up on a table. Shuffle the orange cards and let the child draw and make pairs with the cards on the table. He must give the product before the pair is his.

8.

X	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

There are many ways to work with students using the multiplication facts grid, or table.

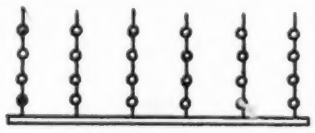
Example for number 8.

a. Point out patterns:
The two row and the two column have the same entries. The diagonal from left to right has the "square" numbers 2 x 2 = 4, 3 x 3 = 9, etc.

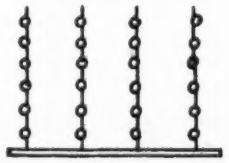
Observe in the 9's column that the digits in the one's place decrease by one and the digits in the ten's place increase by one, and the sum of the digits in the product is equal to 9. (9 x 9 = 81. 81 is the product. 8 + 1 = 9. Now look at the product of 8 x 9. It is 72. 7 + 2 = 9.)

b. The primary purpose of the table is intended to help children commit the multiplication facts to memory.

9. Use the bead frame to illustrate the meaning of the various multiplication combinations such as:



4 x 6 = 24



6 x 4 = 24

10. A commercial game which is considered excellent for the purpose of learning multiplication facts is "The Winning Touch" by the Ideal Company.

"Twin Choice" - Holt, Rinehart, and Winston is also recommended.

11. A book From Fingers to Computers, Franklin Mathematics Series, by Webster Publishing Company contains Napier's Bones, another interesting and "different" approach.

VOLUNTEER - MATHEMATICS

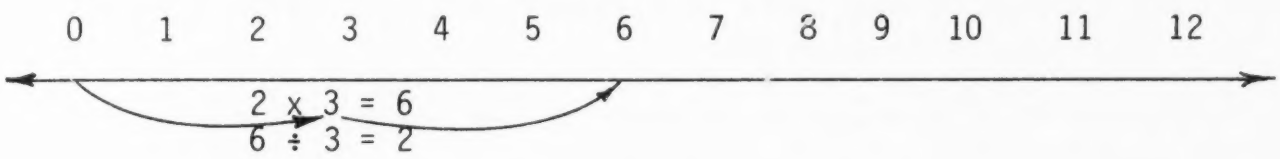
DIVISION FACTS AND THEIR RELATIONSHIP TO MULTIPLICATION

Division is taught as the inverse of multiplication. Many activities from the section on multiplication can be used in reverse where you give a factor and a product and ask for the missing factor. Example:

$$\square \times 4 = 16$$

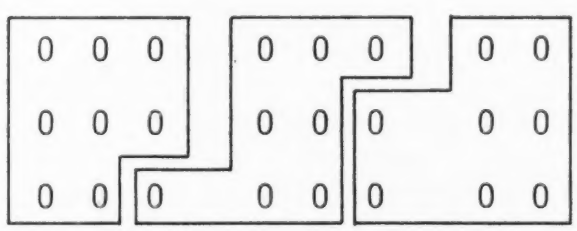
1. Separate many sets of objects into equivalent sets. Have the student tell and/or write the corresponding division fact.

2. Use the number line to help the student see that $2 \times 3 = 6$ and $6 \div 3 = 2$:



Put on the chalkboard:

Suppose you did not know the solution of $24 \div 8 = \square$
Draw the set array:



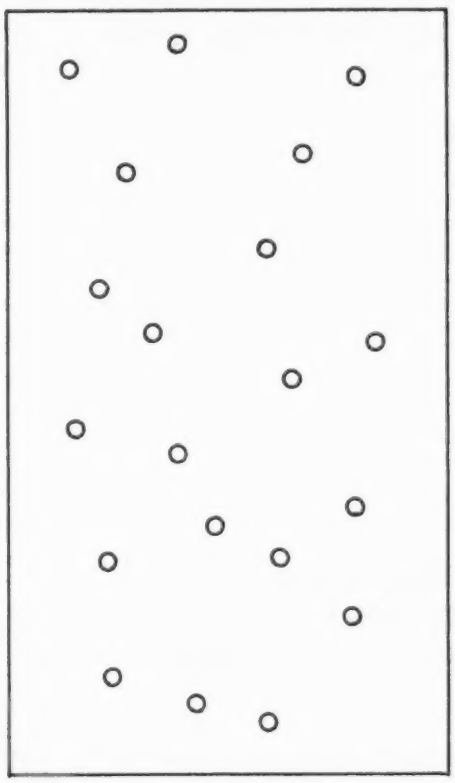
8	24
8	-8
+8	16
24	-8
	8
	-8
	0

Discuss this operation using the chalkboard:

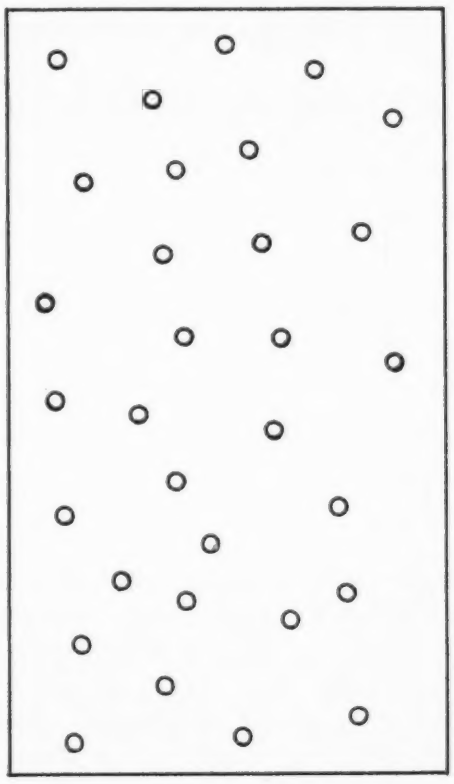
a. 18 - 3 = 15
b. 15 - 3 = 12
c. 12 - 3 = 9
d. 9 - 3 = 6
e. 6 - 3 = 3
f. 3 - 3 = 0

The 3 was subtracted 6 times. In a set of 18 there are 6 sets of 3.

3. Children may use the grouping board (cards will do as well) and chalk to discover division facts:

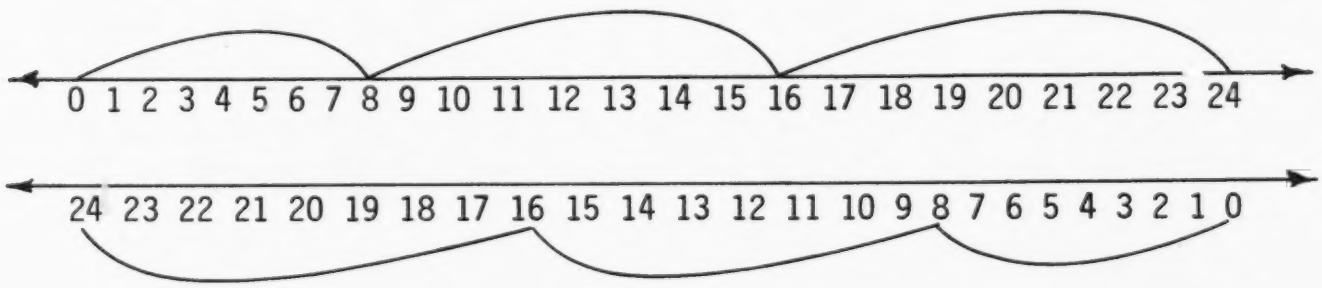


How many dots? _____
How many sets of three? _____
Any left over ? _____
☐ ÷ 3 = ☐ and _____ left over.

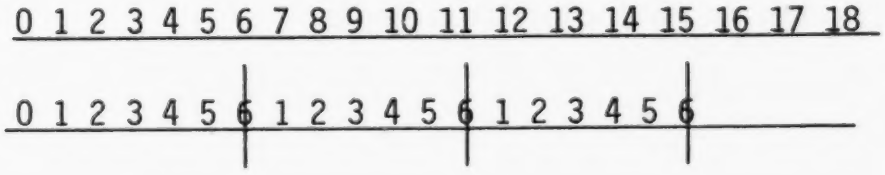


How many dots? _____
How many sets of seven? _____
Any left over? _____
☐ ÷ 7 = ☐ and _____ left over.

4. If children have trouble relating division to number line starting at zero let them use it backwards: 24 - 8 = ☐

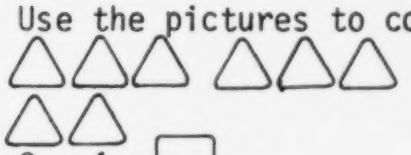


Or let them cut up some number lines into segments, and use them like this:

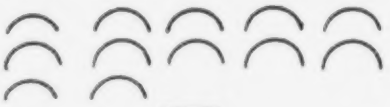


5. Play the game, "Find Your Mate", with the group. Prepare tag board strips with a multiplication equation on one and the division equation on the other. Shuffle all the cards and deal them to the group. Ask the children to line up in two rows. See how quickly they can find their mate. Summarize the findings by having the children in each pair read their equations.

6. Use the pictures to complete the sentences:



8 ÷ 4 = ☐



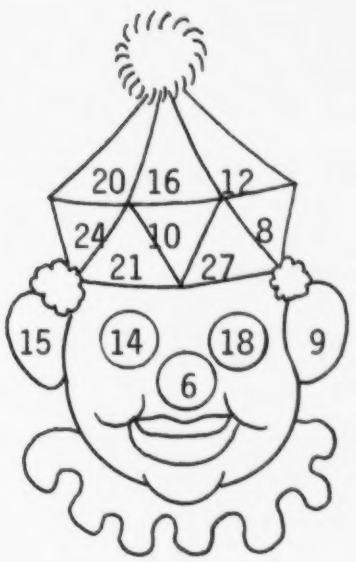
12 ÷ 3 = ☐

18 ÷ 6 = ☐

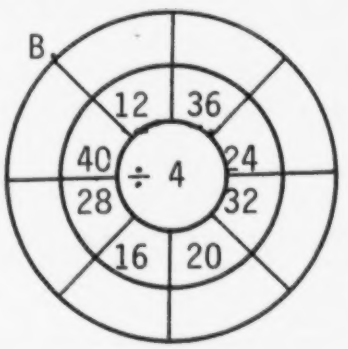
7. Put some simple sample stories on the board or duplicate:
   a. A set of twenty may be separated into how many sets of four?
   b. A frog jumped a distance of eighteen feet on three equal leaps. How long was each leap?
   c. Four sets of six make one set of ☐.
   d. If you had twenty baseball cards and wanted to put them into piles of four, how many piles would you have?

8. Practice on basic facts can be provided by exercises like the following:

A.



B



   a. Make a picture of a clown similar to this. Point to 20, 16, 8, 24, 4, and 12. Have a student divide each by 4 as you point. Then have another divide by 3 as you point to 12, 24, 27, 21, 15, 18, 6, and 9. Have several pupils divide the even numbers by 2.
   b. Divide each product by 4 and put the answer in the outer rim.

9. Using a completed multiplication grid show the children how it can be used for division facts. See section on multiplication facts.

10. Ask the children to use the $>$, $<$, = symbols to express the relation between the numbers represented by the numerals. Have them write the symbol in the circle:

48 ÷ 6	◯	4 x 9
6 x 6	◯	36 ÷ 6
36 ÷ 4	◯	36 ÷ 6
9 x 7	◯	7 x 9
18 ÷ 9	◯	(3 x 1) x 3

11. FACT FAMILY RUMMY

Objective: Students will be able to demonstrate understanding of relation between multiplication and division by building packs of related facts.

Materials: 50 cards on which are written the related multiplication and division facts for 5 x 5 through 9 x 9.

Example: $5 \times 5 = 25$, $25 \div 5 = 5$, $6 \times 5 = 30$, $30 \div 5 = 6$

Rules:

1. The game may be played like Gin Rummy. The object of the game is to be able to lay down all the cards in your hand 1, 2, or 3 at a time.

   1 card - When you are playing on a set built by some other player, such as 42 ÷ 7. This also completes a pack.

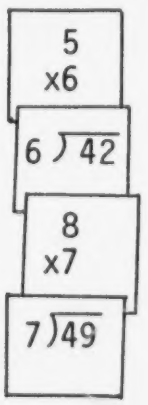
   2 cards - When you have a pack made up of a double, such as 5 x 5, 25 ÷ 5.

   3 cards - When you have a set of three members of fact family, such as 6 x 7, 7 x 6, 42 ÷ 6.

2. The dealer begins by passing out 7 cards to each player. The remainder of the deck is placed face down on the playing board. The top card is turned face up to form the discard pile.

3. The player to the left of the dealer begins the game by either taking a card from the drawing pile or the discard pile.

   Cards in the discard pile are placed in such a way that the fact on each card can be seen. Example:



4. If the player takes a card from the drawing pile, he will see if he can put it with something in his hand to build or complete a pack. Two in doubles and three of a 4-card pack can be laid on the playing board as credit to the person playing.

   If the player wants a card from the discard pile, he must take all the cards on top of it. Then he must lay down that card as part of a set.

5. If someone lays down the wrong cards, he must take them back and lose his turn.

6. If the players go through the drawing pile before the game is over, reshuffle the discard pile and continue.

7. Each player will receive five points for every card on his board, minus five points for every card left in his hand.

OPERATIONS - ADDITION

1. The use of set materials in developing the concept of addition of two-digit numerals is quite important. The skill for adding two-digit numbers can be developed rather quickly; but in order to understand the concept involved, it is essential for them to see what is happening.

   Use number sentences like the following and have the pupils fill the place holders to make a true sentence:

   Example: $25 + 3 = (20 + 5) + 3 = 20 + 8 = \triangle$

   $14 + 5 = (10 + \square) + 5 = 10 + 0 = \triangle$

   $26 + 3 = (20 + \square) + 3 = 20 + 0 = \triangle$

   $13 + 4 = (10 + \square) + 4 = 10 + 0 = \triangle$

   Use an abacus and separate the two numbers with a clothespin or clip to give visual meaning to the problems.

2. To promote discussion and thinking, put equations like the following on the board and ask the children if they are true. By illustrating or manipulation of sets, they should prove them:

   a. $3 + (1 + 2) = 6$
   b. $(3 + 1) + 2 = 6$
   c. $(2 + 4) + 1 = 2 + (4 + 1)$

3. Display two sets whose union would be a set with more than ten members. Tell the class you want to write a number sentence showing the joining of the two sets. Tell them you want to look for a ten. Ask them to suggest ways to partition one of the sets and make a set of 10. Record the number sentence. Put the sets back like they were and ask them to find another way. Follow up the activity with a written exercise such as the following. If necessary, let children use sets:

   $6 + 5 = 6 + (\triangle + \square) = 10 + \square$

   $9 + 7 = (\square + \triangle) + 7 = \square + 10$

4. Put a set of four objects and a set of five objects on the flannel board or wherever it is convenient. Have children identify the number of objects in each set. Frame the sets and have the children identify the number of members in the union. Ask for an equation or number sentence that you can relate to the union of the sets ($4 + 5 = 9$). Ask, then, if someone can use the number line to illustrate that $4 + 5 = 9$. Go back to the board and, having removed the first sets, put a set of five, and then a set of four. Continue as above,using different sets to help the children see that the order in which we join sets and add numbers does not change the final result.

5. After the preceding activity, using the same pairs of equations, use this activity and let the children circle or mark in some way the ones that are true sentences:

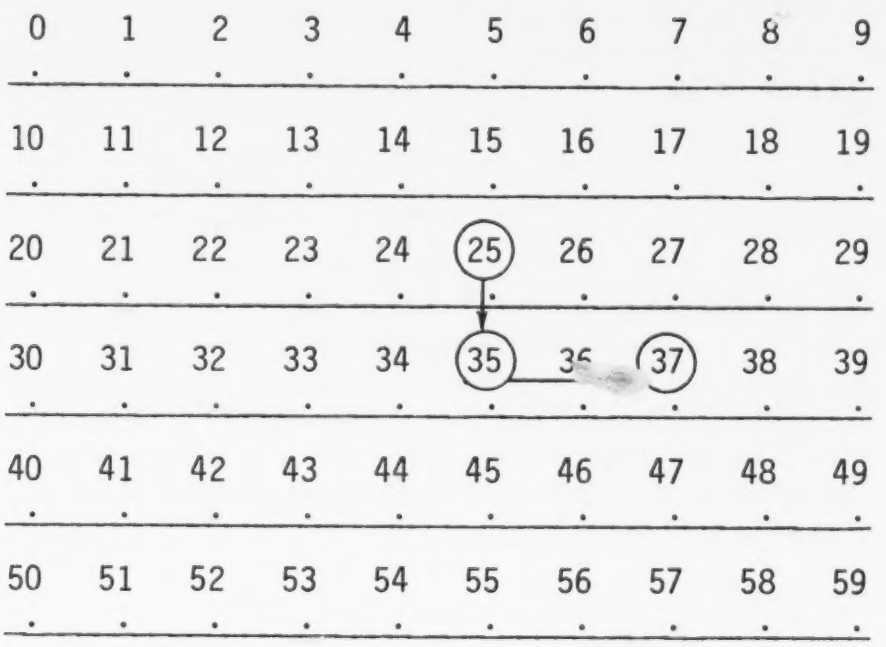
4 + 5 = 5 + 4	6 + 1 = 2 + 6
3 + 2 = 2 + 3	3 + 4 = 4 + 3

If they have difficulty, let them use sets to decide which ones are true.

6. Using three set instead of two, work through an activity similar to number one. Then use the following and ask the children to find the sentences that have the same sum:

(3 + 2) + 1 = ☐
3 + (5 + 1) = ☐
3 + (2 + 1) = ☐
(3 + 5) + 1 = ☐

7. Use a continuous number line, and have the pupils start with the first addend and by renaming move to add the second addend. For example: 25 + 12; start at 25, move down one to add ten and right 2, (10 + 2), to name the sum:



8. Use a number square like the following:

Add →

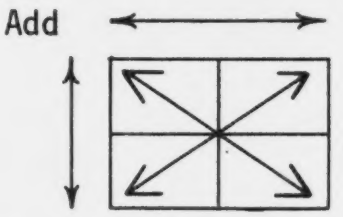
40	8	(48)
30	9	(39)
(70)	(17)	(87)

Numbers in parentheses are to be filled in.

9. Take any four numbers and arrange in a square:

4	3
7	0

Add



Add rows , columns, and diagonals, as shown, to expand the square:

13	11	12	10
7	4	3	7
16	7	9	16
10	11	12	13

Then do the same thing again. Vary it by making the original numbers larger. Note the pattern.

10. Place a rectangle like the following on the floor. Have a student throw two bean bags and add the two numbers:

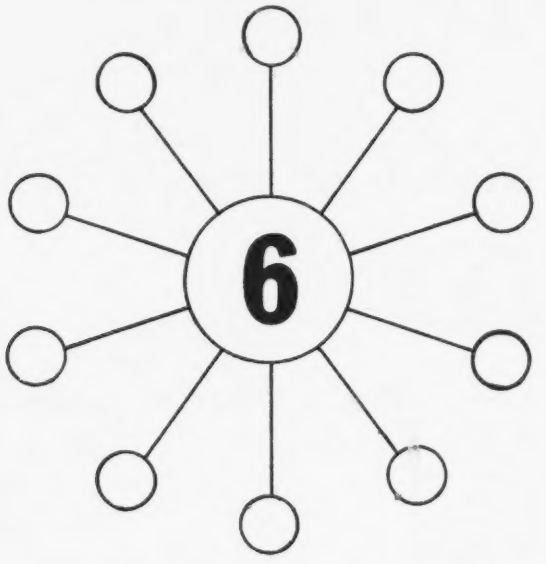
10	20	10
10	50	20
30	20	10

Change the activity by changing the numbers.

11. "GOAL" - Students work in pairs; each pair has paper and pencil and two cubes in a cup. The faces of each cube are numbered from 0 through 5. To play the game, each student takes a turn to throw the cubes from the cup. He then, adds the numbers shown by the cubes and writes an equation with his sum and his score. Example: if a student with a score of 19 throws cubes showing five and four, he writes 19 + 9 = 28. The other students would check his work. Set a goal of 95 or whatever, and the first one to reach the goal wins. (See packet for cubes.)

12. For drill in addition of larger numbers, several children can play a game where they draw numeral cards with two-digit numerals on them. Each takes a turn drawing a card and keeps a cumulative score by adding the card drawn to his previous score. The first one to reach a designated number wins. Time and the skill of the players will determine the size of the winning number. A spinner could also be used. To combine drill in addition and subtraction, use cards with subtraction problems, such as 65 - 32 = N, on them. The players must solve the equation and then add the solution to his score. A counting frame or hundred board should be handy for checking computation.

13. Give the children a copy of the following or put it on the chalkboard. Instruct them to place the numbers 1 to 11 in the eleven circles so that the three numbers in each line add to 18:



14. Play a game similar to "Simon Says." Have the children use their fingers to show the numerals that represent the answers. One hand would be ones, the other tens. Say "Simon Says": show the numeral for 0 + 3. Now add 10. Subtract (2 + 3). Divide by 4. Multiply by 3. . . .Continue making the problems more difficult.

15. Complete each number pattern until you have nine digits on the right-hand side of the equation:

a. 9 + 1 = 10
   99 + 1 = 100

b. 8 + 2 = 10
   98 + 2 = 100

c. 3 x 2 = 1 + 2 + 3
   5 x 2 = 1 + 2 + 3 + 4

16. Fill the blanks to fit the pattern:

a. 2, 5, 4, 7, 6, (9), (8), (11)
b. 4, 9, 19, 39, (79), (159), (339)
c. 6, 16, 26, ___, ___, ___

17. To find the simplest numeral for 12345, 9078, 8665, a place value grid is shown:

T	Th	Th	H	T	O	
1		2	3	4	5	
		9	0	7	8	addends
		8	6	6	5	
3		0	0	8	8	sum

Place these numerals correctly on a place value gric and add:
1. 173,425 2. 609,34 3. 72,001

18. Expanded notation for addition:

Step I
$$\begin{array}{r} 54 = 50 + 4 \\ \underline{+39} = \underline{30 + 9} \end{array}$$

Step II
$$\begin{array}{r} 54 = 50 + 4 \\ \underline{+39} = \underline{30 + 9} \\ 80 + 13 \end{array}$$

Step III
$$\begin{array}{r} 54 = 50 + 4 \\ \underline{+39} = \underline{30 + 9} \\ 80 + (10 + 3) = 90 + 3 = 93 \end{array}$$

19. Finish these addition tables. See if you find some number patterns up or down, diagonally, or across:

+	44	54	64	76
8				
16				
24				
32				

+	7	11	15
15			
85			
95			

20. What is the sum?

a. $8 + 0 = \square$
b. $39 + 0 = \square$
c. $1492 + 0 = \square$
d. $19 + \square = 19$
e. $1965 = \square + 0$

21. Mark the true sentences:

a. $(6 + 2) - 8 = 0$
b. $(9 + 7) - 7 = 3 + 6$
c. $12 + 24 = 24 + 12$
d. $76 + 35 = (70 + 30) + (6 + 5)$

22. Some of the properties studied earlier are reviewed below. Elicit from the pupils if the statements are always true, or never true. Have them give examples:

    a. If 0 is added to any number, the result is that number. (Always true.)
    b. If the order of adding two whole numbers is changed, the sum is unchanged. (Always true.)
    c. If 0 is subtracted from a whole number, the result is that whole number. (Always true.)
    d. If two whole numbers are subtracted, the result is a whole number. (Sometimes true.)

23. Find n so that each mathematical sentence is true:

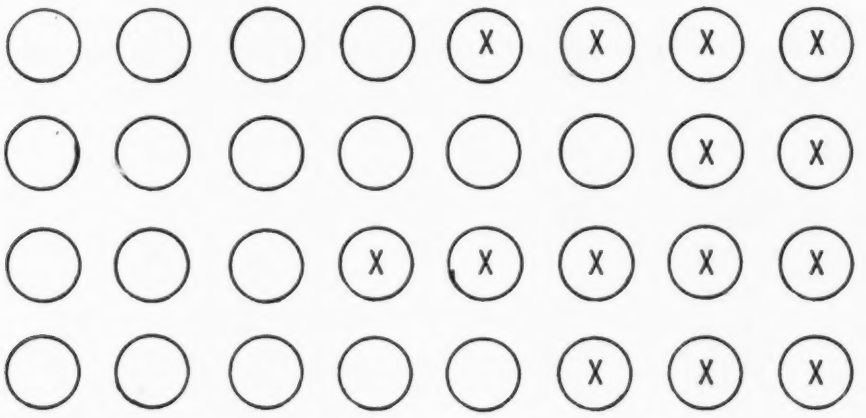
a.	$6 + n = 6 + 9$	Answers (9)
b.	$12 + n = 14 + 12$	(14)
c.	$(8 + 8) + n = 8 + (9 + 4)$	(5)
d.	$(3 + 5) + (n + 2) = (6 + 2) + (3 + 5)$	(6)

24. Find what number n represents so that each mathematical sentence is true. There may be more than one answer, one answer, or no answer:

a.	$(3 + 2) + 8 = n$	(13)
b.	$(3 + 2) + n = 8$	(3)
c.	$(3 + 2) - n = 8$	(no answer)
d.	$1 + n = n + 1$	(more than one answer)
e.	$(7 + 3) + n = 7 + (3 + n)$	(more than one answer)

OPERATIONS - SUBTRACTION

1. Have the children copy the following illustrations and put the correct equations at the side of each:



2. Questions for discussion:
   a. How can you find the difference in a subtraction fact if you forget it?
   b. How do you use a table to solve 17 - 9 = n?
   c. How can ten help you add and subtract?

3. Expanded notation for subtraction:

Step I
$$\begin{array}{r} 54 = 50 + 4 \\ \underline{-39} = \underline{30 + 9} \end{array}$$

Step II
$$\begin{array}{r} \quad\quad 40 \quad 14 \\ 54 = \not{5}\not{0} + \not{4} \\ \underline{-39} = \underline{30 + \ 9} \end{array}$$

Step III
$$\begin{array}{r} 54 = 40 + 14 \\ \underline{-39} = \underline{30 + \ 9} \\ 10 + \ 5 = 15 \end{array}$$

4. Which is the best way to rename the sum?

$$\begin{array}{l} 604 \\ \underline{-126} \end{array} \quad \left\{ \begin{array}{l} 600 + 4 \\ 500 + 100 + 4 \\ 60 \text{ tens} + 4 \end{array} \right. \quad \begin{array}{l} 59 \text{ tens} + 14 \\ 500 + 90 + 14 \end{array}$$

5. Use exercises such as the following to help children learn to think in terms of finding how many needed:

$3 + \square = 6$ $\qquad\qquad$ $5 + \square = 9$

Perhaps a game could be devised where a leader says "Have 4, need 7, how many more?" Let him choose someone to answer. If the answer is correct, the one who responded gives one, and so on. Or use flash cards with these exercises on them.

6. This activity suggests a method for subtraction by parts:

18 - 7 = ☐	15 - 9 = ☐
(10 + 8) - 7 = ☐	(5 + 10) - 9 = ☐
10 + (8 - 7) = ☐	5 + (10 - 9) = ☐
10 + 1 = 11	5 + 1 = 6

7. Find an example that is the key to each group of four related examples. Solve it and then use it to determine the missing numerals:

a.

357	234	357	***
-***	+***	-234	+234
234	357	***	357

b.

256	431	***	431
+***	-***	+256	-256
431	256	431	***

8. GAME - Changing Numbers - Show a bundle number in a pocket chart and ask pupils to change it so that it will appear as it might if a ten or (hundred) were renamed. The illustration below shows how 182 would be shown first, and how it should be changed by pupils in borrowing a ten:

a. ~~1111~~ ϕ ϕ ϕ ϕ ϕ ϕ ϕ ϕ 11

b. ~~1111~~ ϕ ϕ ϕ ϕ ϕ ϕ ϕ 11 11 11 11 11 11

As a variation, write a number on the board and show it with bundle numbers in a pocket chart incorrectly. Pupils are to tell what is wrong and correct it.

For example, if the number 357 were shown incorrectly with bundle numbers as below, pupils should read the number as 347 and change it by inserting another ten bundle, reading it then as 356.

~~1111~~ ~~1111~~ ~~1111~~ ϕ ϕ ϕ ϕ 11 11 11 1

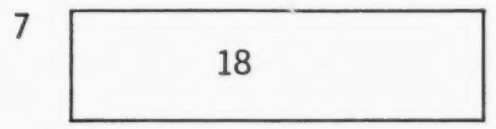
OPERATIONS - MULTIPLICATION

1. Use exercises like the following to help children see the patterns in multiplying by 10's, 100's, 1,000's, etc.:

   a. $10 \times 2 = 20$ $\qquad$ $10 \times 4 = 40$
      $10 \times 3 = 30$ $\qquad$ $10 \times 5 = \square$

   b. $100 \times 2 = 200$ $\qquad$ $100 \times 4 = 400$
      $100 \times 3 = 300$ $\qquad$ $100 \times 5 = \square$

2. Discuss the following with the children. To find the product of 7 and 18, think of a 7 x 18 array:



   Separate it into two arrays showing products already known. For example:

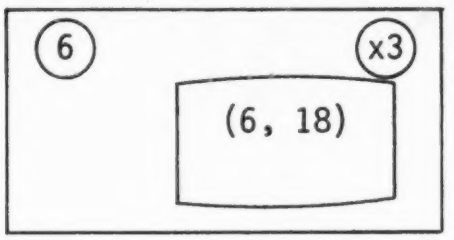
   Find the products separately and add them to get the number of elements in the 7 x 18 array:

$$\begin{aligned} 7 \times 18 &= 7 \times (10 + 8) \\ &= (7 \times 10) + (7 \times 8) \\ &= 70 + 56 \\ &= 126 \end{aligned}$$

3. Have the children complete the following:

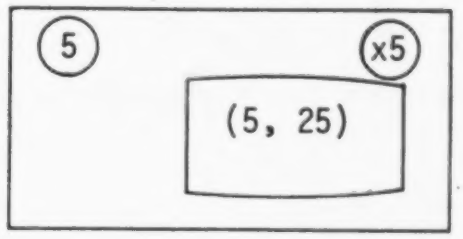
   a. $7 \times 26 = 7 \times (20 + 6) = (7 \times 20) + (__ \times __)$
   b. $5 \times 34 = 5 \times (30 + 4) = (__ \times __) + (__ \times __)$
   c. $6 \times 28 = 6 \times (__ + 8) = (__ \times 20) + (__ \times 8)$
   d. $3 \times 162 = 3 \times (100 + __ + 2) = (3 \times 100) + (3 + 60) + (3 \times __)$
   e. $4 \times 147 = 4 \times (100 + 40 + 7) = 4 \times 100 + 4 \times 40 + 4 \times 7 = 400 + __ + __$

4. Study each function TV, then complete the ordered pair:



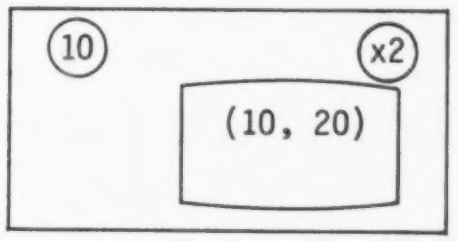
   (6,a), (8,b), (11,c),
   (14,d), (19,e), (23,f)

   a = 18, b = 24, c = 33,
   d = 42, 3 = 57, f = 69



(5,g), (0,h), (8,i),
(1,j), (12,k), (17,e)

g = 25, h = 0, i = 40,
j = 5, k= 60, l =85



(10,m), 8,n), (3,s),
(11,p), (15,g), (22,r)

m = 20, n = 16, s = 6,
p = 22, q = 30, r = 44

5. Use the associative property when one of the factors is a multiple of 10, 100, and so on:

$$\begin{aligned} 4 \times 20 &= 4 \times (2 \times 10) \\ &= (4 \times 2) \times 10 \\ &= 8 \times 10 \\ &= 80 \end{aligned}$$

6. Use the distributive property when one of the factors is greater than 10:

$$\begin{aligned} 6 \times 43 &= 6 \times (40 + 3) \\ &= (6 \times 40) + (6 \times 3) \\ &= 240 + 18 \\ &= 258 \end{aligned}$$

7. Use the distributive property when both of the factors are greater than 10:

$$\begin{aligned} 26 \times 43 &= (20 + 6) \times 43 \\ &= (20 \times 43) + (6 \times 43) \\ &= 860 + 258 \\ &= 1118 \end{aligned}$$

8. Use the associative or distributive property to name the products:

1. 5 x 30 (150)
2. 5 x 50 (250)
3. 4 x 86 (344)
4. 7 x 79 (553)
5. 5 x 243 (1215)
6. 7 x 6 x 8 (336)
7. 28 x 43 (1204)
8. 46 x 53 (2438)

9. Finish this multiplication chart. Add the numbers in each set and multiply the sums. Add the products in the grid. Compare the sums:

		Set A	
		5	3
Set B	2	10	6
	4	20	12
	6	30	18

5 + 3 = 8 2 + 4 + 6 = 12

8 x 12 = 96

10 + 20 + 30 + 6 + 12 + 18

Make up one of your own and see if the same pattern occurs.

10. Complete the number patterns below:

X	3
200	
30	
3	
233	

X	3
100	
20	
2	

X	5
100	
10	
1	

11. Write an example and show each partial product using example for the exercises:

$$\begin{array}{r} 357 \\ \times\ 4 \\ \hline 28 \\ 200 \\ 1200 \\ \hline 1428 \end{array} \quad \begin{array}{l} \\ \\ (4 \times 7) \\ (4 \times 50) \\ (300 \times 4) \\ \\ \end{array}$$

OPERATIONS - DIVISION

1. To find how many sets of 7 may be formed from a set of 31, you may think like this:

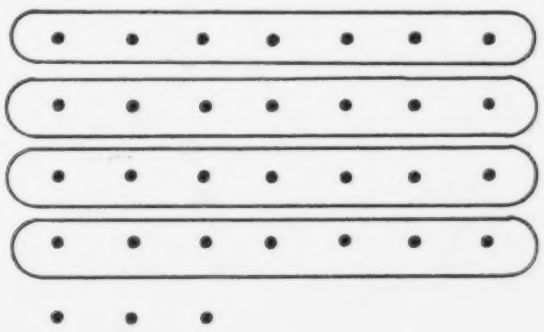
   $4 \times 7 = 28$ $\qquad\qquad$ $5 \times 7 = 35$

   31 is between 28 and 35

   There are more than 4 sets of 7 in 31, but not enough for 5 sets of 7.

   $31 = (4 \times 7) + 3$

   A set of 31 forms 4 sets of 7 with a remainder of 3.



2. Complete the sentences:

   a. A set of 14 forms 4 sets of 3 with a remainder of ___? (2)
   b. A set of 25 forms 6 sets of 4 with a remainder of ___? (1)
   c. A set of 23 forms 3 sets of 7 with a remainder of ___? (2)
   d. A set of 38 forms 4 sets of 8 with a remainder of ___? (6)
   e. A set of 47 forms 9 sets of 5 with a remainder of ___? (2)

   Complete this chart:

Size of Set	34	26	25	20	28	30	32	29	35
Number of Subsets	6	4	3	c	e	q	6	k	m
Size of Each Subset	5	6	8	6	5	4	i	3	4
Number of Objects Remaining	4	a	b	d	f	h	j	L	n

   (c = 3, e = 5, q = 7, k = 9, m = 8, i = 5, a = 2, b = 1, d = 2, f = 3, h = 2, j = 2, L = 2, n = 3)

3. Name the missing factor and the quotient in each pair of equations:

   a. $n \times 10 = 70$
      $70 \div 10 = n$

   b. $n \times 40 = 80$
      $80 \div 40 = n$

   c. $n \times 30 = 60$
      $60 \div 30 = n$

   d. $n \times 50 = 100$
      $100 \div 50 = n$

4. Solve each division equation. Then multiply the dividend and divisor by 10, and name the quotient. Check by multiplication. Example:

   $15 \div 3 = 5 \quad 5 \times 30 = 150 \quad 150 \div 30 = 5$

   a. $16 \div 4 = \square$
   b. $24 \div 8 = \square$
   c. $72 \div 9 = \square$
   d. $54 \div 6 = \square$
   e. $32 \div 8 = \square$
   f. $45 \div 5 = \square$

5. Game - "Find Your Way Out"
   Follow the arrows and divide by 3:

30	600	24	90	36
240	900	360	12	18
180	60	~~300~~	48	~~120~~

6. Fill in the frames; find the solutions. Compare the answers:

   $(40 + 8) \div 8 = (40 \div 8) + (8 \div 8)$
   $= \triangle + \triangledown$
   $= \square$

   $8\overline{)\ 48} = 8\overline{)40 + 8}$
   $\qquad\qquad 5 + \triangle = \square$

7. Some children might like to try the subtractive method of division. Begin by using a simple problem and objects. State a problem such as 6 cookies. Give away 2 at a time. How many people get cookies? Actually give away 6 things by 2's to see what happens. Record each time 2 are given away. 6-2 = 4-2 = 2-2 =0. Ask the children if they could express what happened in another way; lead them to $6 \div 2 = 3$. Then continue to develop the idea with large numbers, such as:

$$\begin{array}{r|l} 6\overline{)\ 72} & \\ \underline{60} & 10 \\ 12 & \\ \underline{12} & \underline{2} \\ & 12 \end{array}$$

or

$$\begin{array}{r} \underline{12} \\ 2 \\ \hline 10 \\ 6\overline{)\ 72} \\ \underline{60} = (10 \times 6) \\ 12 \\ \underline{12} = (2 \times 6) \\ 0 \end{array}$$

8. Other methods of division:

a. EQUATIONS

$56 \div 4 = (40 + 16) \div 4$
$= (40 \div 4) + (16 \div 4)$
$= 10 + 4$
$= 14$

WORKING FORM

$4\overline{)56}$
40 | 10
16
16 | 4
14

check
$10 \times 4 = 40$
$4 \times 4 = 16$
56

b. EXPANDED NUMERALS

$4\overline{)56} = 4\overline{)40 + 16}$ (10 + 4)

SHORTER WORKING FORM

$4\overline{)56}$ (14)
40
16
16

9. From the set (10, 20, 30, 40, 50, 60, 70, 80, 90), choose the largest number that will make the inequality sentence true. Write this number in the circle. Then write the other missing digits to complete the dividing. For example:

a.
$3\overline{)54}$ (18)
-30
24
-24
0

(10) $\times 3 < 54$

b.
$2\overline{)85}$ (f)
$-a$
c
$-d$
e

(b) $\times 2 < 85$
(a=80, b=40, c=5, d=4, e=1, f=42)

c.
$4\overline{)132}$ (f)
$-a$
c
$-d$
e

(b) $\times 4 < 132$
(a=120, b=30, c=12, d=12, e=9, f=33)

d.
$6\overline{)263}$ (f)
$-a$
c
$-d$
e

(b) $\times 6 < 263$
(a=240, b=40, c=23, d=18, e=5, f=43)

10. Replace the frame in each sentence with the greatest whole number that will make a true statement. Then divide to find the quotient and remainder. Show the subtraction. Example:

$\boxed{8} \times 10 < 87$

$10\overline{)87}$ (8 r7)
80
7

a. $\square \times 20 < 44$ $20\overline{)44}$
b. $\square \times 30 < 66$ $30\overline{)66}$
c. $\square \times 70 < 140$ $70\overline{)140}$
d. $\square \times 50 < 268$ $50\overline{)268}$
e. $\square \times 60 < 467$ $60\overline{)467}$

11. Complete each division and the checking sentence:

$$60\overline{)290}\quad 4,\ 240,\ 50 \qquad 80\overline{)523}\quad 6,\ 480,\ 43$$

$290 = (60 \times \square) + 50$
$290 = (240 + 50)$
$290 = 290$

$523 = (80 \times 6) + \square$
$523 = (480 + 43)$
$523 = 523$

12. To help children develop skill in division, allow them to do exercises such as the following. Find the largest whole number that will make the following sentences true:

a. $n \times 30 < 287$ (9)
b. $n \times 40 < 258$ (6)
c. $b \times 70 < 438$ (6)
d. $c \times 60 < 557$ (9)
e. $r \times 80 < 416$ (5)

Use the above work to find the quotient and remainder of:

a. $287 \div 30$ $\quad 30\overline{)287}$, 270, 17 $\quad 9$ ← This is the largest whole number that will make $n \times 30 < 287$ true.

$287 = 30 \times 9 + 17$

b. $258 \div 40$
c. $438 \div 70$
d. $557 \div 60$
e. $416 \div 80$

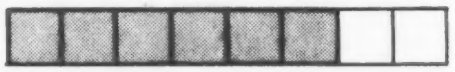
VOLUNTEER - MATHEMATICS

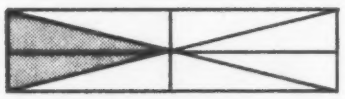
KNOWLEDGE AND MEANING OF FRACTIONS

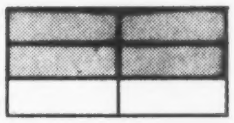
1. Use pieces of construction paper cut into geometric shapes to help children discover:
   a. How many ways a particular shape can be separated to show halves. Use a variety of shapes; circles, squares, etc.
   b. Which fractional parts are larger or smaller? Let the child observe that the larger the number of parts, the smaller the individual parts.

2. Using figures A, B, C, D, E, and F, complete the following table. Then use the number pair to name the shaded part:

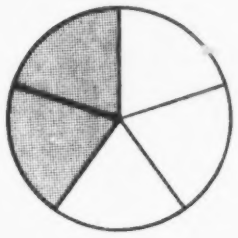
Figure	Parts Shaded	Congruent Parts in Unit
A	(6)	(8)
B	(2)	(8)
C	(4)	(6)
D	(6)	(10)
E	(2)	(5)
F	(2)	(6)

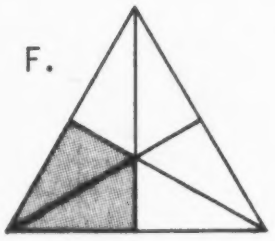
A. 

B. 

C. 

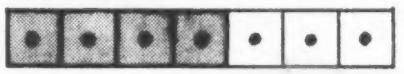
D.

E. 

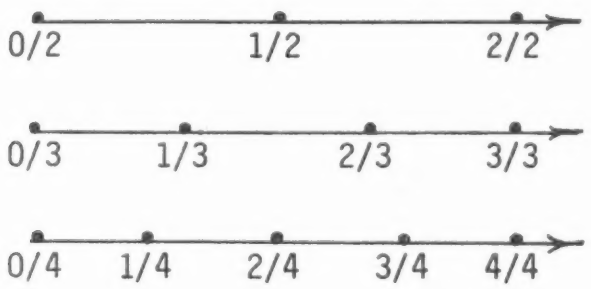
F. 

3. We use a pair of counting numbers to name fractional numbers. If 1 of a set of 2 shapes is colored, we say one-half of the shape is colored and write $\frac{1}{2}$. What fractional numerals describe the colored part of these sets?

A. B.

C. D. 

4. Use these number lines to help you replace each ◯ with $<$ or $>$ to make a true statement:

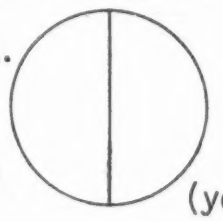


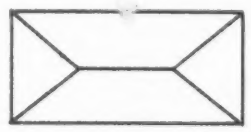
a. 1/4 ◯ 1/3
b. 1/2 ◯ 1/3
c. 2/3 ◯ 3/4
d. 1/2 ◯ 2/3
e. 3/4 ◯ 1/2

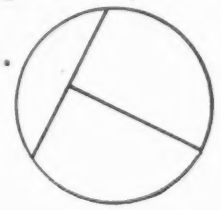
5. Copy and fill in each blank with the symbol $>$, $<$, or =. Refer to the numberline if necessary:

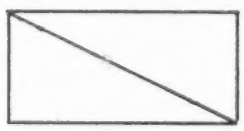
a. 3/3 (>) 1/3
b. 2 (>) 4/3
c. 4/4 (=) 1
d. 4/5 (>) 2/5
e. 1 (<) 2
f. 3/3 (=) 2/2
g. 2 (>) 7/4
h. 1/2 (<) 1

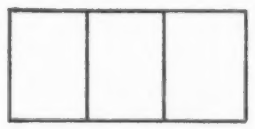
6. Which figures below have been partitioned into congruent parts?

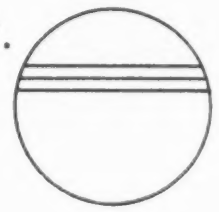
A.  (yes)

B.  (no)

C.  (no)

D.  (yes)

E.  (yes)

F.  (no)

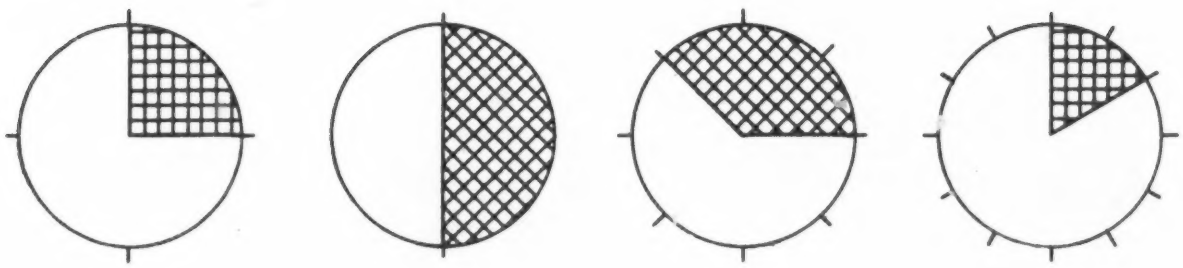
In the picture below the segment AB is 1 unit long.

(Continued on next page.)

VOLUNTEER - MATHEMATICS

a. On the segment, the unit is separated into _____congruent segments. (8)
b. Each small segment is _____ of the unit segment. (1/8)
c. The measure of AB is _____. (1)
d. What fraction would correspond to C? (4/8)
e. The fraction _____ corresponds to point D. (10/8)
f. What is another name for 10/8?
g. Give other names for: 7/6, 14/8, 33/4, 15/9, 24/9

MODELS FOR NAMING FRACTIONAL PARTS



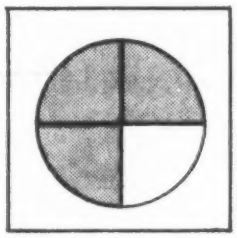
A set of these may be made with as many fractional parts as deemed necessary. Practice in recognition follows: have the pupil look at and guesstimate the fractional name of the shaded part.

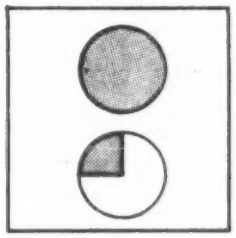
Pupil could also match those that represent the same number - 2/12 and 1/6, for instance.

They could also order them in sequence by writing the smallest to the largest.

Fraction Bingo can be played with this set also. Write fractions on score cards and fractional parts on flash cards.

1/2	1½	1¼
1/4	Free	1
1/3	2/3	3/4



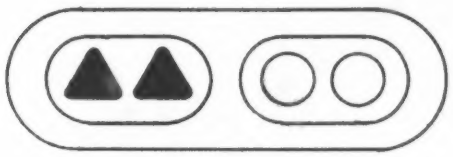


Hold up the fractional parts cards one at a time, and if the player has that number name on his bingo card, he covers it. He calls out "Bingo" when he has covered a diagonal or a row or column.

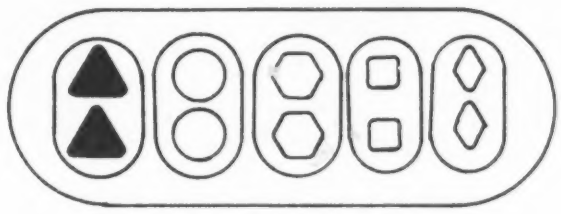
VOLUNTEER - MATHEMATICS

EQUIVALENT FRACTIONS

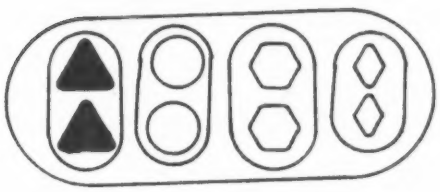
1. Study the sets and solve the equations:



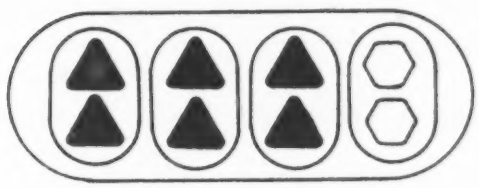
1. $\frac{1}{2} = \frac{a}{4}$
   (a = 2)



2. $\frac{1}{5} = \frac{c}{10}$
   (c = 2)

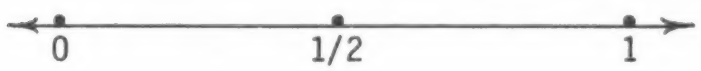


3. $\frac{1}{4} = \frac{e}{8}$
   (e = 2)



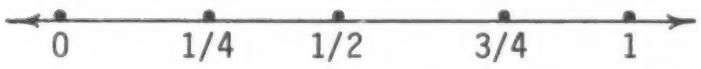
4. $\frac{3}{4} = \frac{i}{8}$
   (i = 6)

2. Study the number lines and solve the equations:



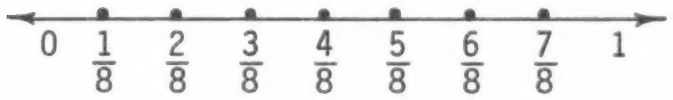
1. $\frac{1}{2} = \frac{a}{8}$
   (a = 4)

2. $\frac{2}{2} = \frac{b}{8}$
   (b = 8)



3. $\frac{1}{4} = \frac{c}{8}$
   (c = 2)

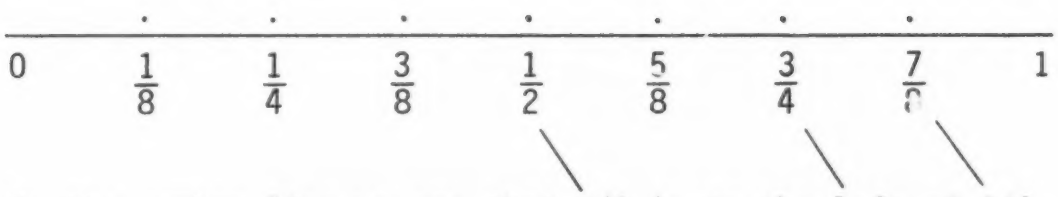
4. $\frac{3}{4} = \frac{d}{8}$
   (d = 6)



5. $\frac{2}{2} = \frac{e}{4}$
   (e = 4)

6. $\frac{4}{4} = \frac{f}{8} = \frac{h}{2}$
   (f = 8h = 2)

3. Discuss with students:



On the number line we see that 1/2 is to the left of 3/4, $1/2 < 3/4$; 7/8 is to the right of 3/4, $7/8 > 3/4$.

Find the LCM of the denominator in each pair. Write the fractions for each pair with the LCM as the denominator. Then replace each◯ by $<$, $>$, or =.

4. Use the pictures to solve each set of equations:

1					
1/3		1/3		1/3	
1/6	1/6	1/6	1/6	1/6	1/6
1/2		1/2			

a. $1/3 = \frac{\square}{6}$

b. $2/3 = \frac{\square}{6}$

c. $1/2 = \frac{\square}{6}$

d. $1 = \frac{\square}{6}$

1									
1/5		1/5		1/5		1/5		1/5	
1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10
1/2					1/2				

a. $1/5 = \frac{\square\square}{10}$

b. $2/5 = \frac{\square\square}{10}$

c. $1/2 = \frac{\square\square}{10}$

d. $1 = \frac{\square\square}{10}$

5. Use exercises like the following for oral discussion:

   Jane said, "One-half of the pencils are blue."
   Sam said, "Two-fourths of the pencils are blue."
   Are they talking about the same number of pencils?

   Complete each set of names for the same fractional number:

   a. $\frac{1}{2}$, $\frac{\square}{4}$, $\frac{\square}{8}$, $\frac{\square}{6}$, $\frac{\square}{12}$

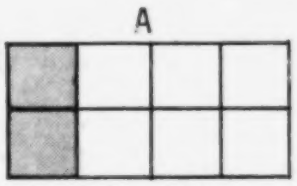
   b. $\frac{3}{4}$, $\frac{\square}{8}$, $\frac{\square}{12}$

   c. $\frac{1}{3}$, $\frac{\square}{6}$, $\frac{\square}{12}$

   d. $\frac{5}{6}$, $\frac{\square}{12}$, $\frac{\square}{18}$

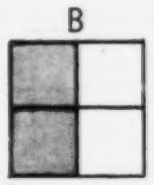
6. Ask children <u>orally</u> to give two fractions for the colored parts:

   A



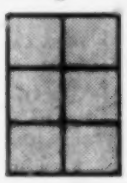
   2/8, 1/4

   B



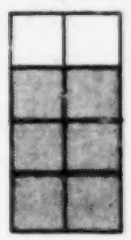
   2/4, 1/2

   C



   $\frac{6}{6}$, $\frac{3}{3}$, $\frac{2}{2}$, $\frac{1}{1}$

   D



   6/8, 3/4

7. To rename a fractional number so that it has a greater denominator, we <u>multiply</u> both numerator and denominator by the same factor. Example:

   $1/2 = 1 \times 1/2 = \frac{2 \times 1}{2 \times 2} = \frac{2}{4}$

   a. $\frac{1 \times n = 2}{4 \times n = 8}$

   b. $\frac{1 \times d = 4}{2 \times d = 8}$

   c. $\frac{5 \times c = 10}{6 \times c = 12}$

   d. $\frac{3 \times a = 6}{5 \times a = 10}$

8. You can rename 2/3 and 3/4 as shown below:

   2/3 = 4/6 = 8/12 = 10/15 = 12/18 = 14/21 = 16/24 = 18/27. . .
   3/4 = 6/8 = 9/12 = 12/16 = 15/20 = 18/24 = 21/28 = 24/32 = 27/36. . .

   Could both 2/3 and 3/4 be named with 12 as a denominator? With 24 as a denominator? The set of numbers below is called the set of common denominators of 2/3 and 3/4.

   (12, 24, 36, 48. . .)

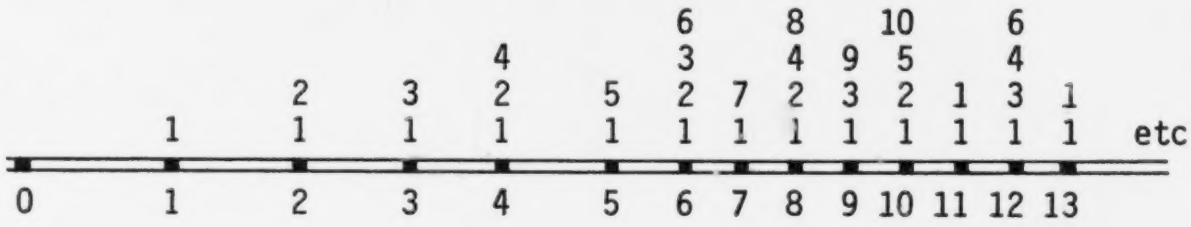
   Does this set have a least member? That member is called the least common denominator of 2/3 and 3/4.

   Consider the denominators of 2/3 and 3/4. What is the least common multiple of 3 and 4? (12) Is it the same as the least common denominator found above? (yes)

   Find the least common denominator for each set:

   a. (1/2, 2/3) (6)
   b. (3/4, 1/2) (8)
   c. (1/5, 1/2, 3/10) (10)
   d. (7/8, 1/4) (8)
   e. (1/3, 3/4) (12)

9. Draw a number line from 0 to 36. Ask the students to name pairs of factors for various numbers and record as follows:



   Ask questions, such as what factors are common to 6 and 12 or 4 and 8.

10. Use the above number line to answer the following; name the factors of:

    a. 4
    b. 6
    c. 8
    d. 9
    e. 12
    f. 13

11. Use the number line to name the numbers less than 25 :

    a. that have 2 and 10 as common factors (0, 20, 10)
    b. that have 4 and 6 as common factors (0, 12, 24)
    c. that have 3 and 8 as common factors

12. Use the number line to name the common factors of:

    a. 8 & 24
    b. 12 & 24
    c. 16 & 24
    d. 20 & 24
    e. 18 & 20

13. What is n, so each sentence is true?

    a. $\frac{2}{8} = \frac{n}{4}$
    b. $\frac{6}{8} = \frac{n}{4}$
    c. $\frac{12}{16} = \frac{\square}{4}$
    d. $\frac{9}{12} = \frac{\square}{4}$
    e. $\frac{6}{10} = \frac{\square}{5}$
    f. $\frac{3}{15} = \frac{\square}{5}$

GAMES

Title: Fic-Fac-Foe-I

Topic: Equivalent Fractions

Objective: Drill generating fraction classes

Materials: Pencil and paper

Procedure:

1. Draw the traditional Tic-Tac-Toe grid.

2. Each player selects a fraction to be used as <u>his</u> mark. He will use this fraction and its equivalents to mark his positions.

   Example: X = 1/2 Y = 1/3

1/3	4/8	3/9
2/6	1/2	2/4
3/6	4/12	5/10

3. For each successive game, new fractions must be used. (Attempt to make it difficult for your partner to recognize your fraction when <u>you</u> get very good at it.)

VOLUNTEER - MATHEMATICS

Card Games: Equivalent Fractions

FRACTION RUMMY

Purpose: To give practice in recognizing equivalent fractions.

Equipment: One deck of fraction cards.

Number of Players: Two - four.

Play: Deal six cards to each player. Place the remainder of the cards face down on the center of the table. The object is to collect sets of three cards with equivalent fractions (three of a kind) or sets of three cards such as 1/4, 2/4, 3/4 (a run). Equivalent fractions may be substituted to make up a "run". The first player draws a card. If he keeps it, he must discard a card (which he places face up in the center of the table) from his hand. If he doesn't have a set, he places it face up before him. A player may pick up the top card which has been discarded instead of drawing a card. The first player to have no cards is the winner.

Caution: Players of about equal ability should be grouped together. It might speed up play if a chart showing equivalent fractions is used by each player.

Variations: A. A player may decrease the number of cards in his hand by playing on the sets of other players.

Example: A player has put down a run consisting of 2/5, 3/5, 4/5. At his turn, another player places 1/5 from his hand on the other player's run. He could have played on it with any fraction equivalent to 1/5.

B. This game may be played as gin rummy is played.

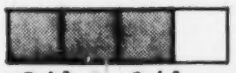
Cards included in set:

1/2, 2/2, 1/3, 2/3, 3/3, 1/4, 2/4, 3/4, 4/4, 1/5, 2/5, 3/5, 4/5, 5/5, 1/6, 2/6, 3/6, 4/6, 5/6, 6/6, 1/8, 2/8, 3/8, 4/8, 5/8, 6/8, 7/8, 8/8, 3/9, 6/9, 2/10, 4/10, 6/10, 8/10, 2/12, 3/12, 4/12, 8/12, 9/12, 10/12, 3/15, 6/15, 9/15, 12/15, 2/16, 4/16, 10/16, 12/16, 14/16, 3/18, 15/18, 4/20, 8/20, 12/20, 16/20, 3/24, 4/24, 9/24, 15/24, 20/24, 21/24, 4/32, 12/32, 20/32, 28/32

OPERATION WITH FRACTIONS - ADDITION AND SUBTRACTION

1. Complete each equation:

a.
1/3 + 1/3 = ☐

b. 
2/4 + 1/4 = ☐

c.
6/7 + 1/7 = 7/7 = ☐

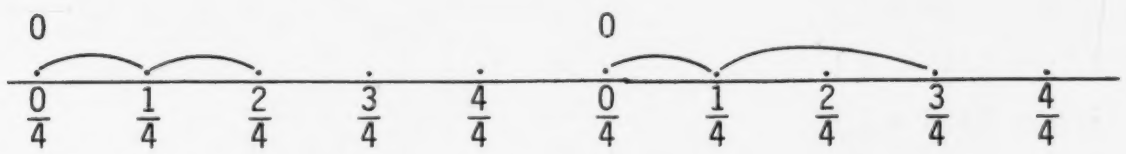
d.
1/4 + 1/4 = △ = ☐

2. Illustrate, using rectangles, these equations and find the solution:

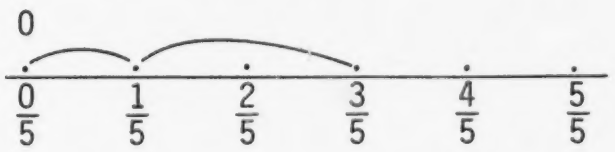
1/2 + 1/2 = ☐ 2/3 + 1/3 = ☐ 2/4 + 1/4 = ☐

4/6 + 2/6 = ☐ 3/6 + 2/6 = ☐ 1/10 + 3/10 = ☐

3. Add fractional numbers on the number line:



1/4 + 1/4 = 2/4 1/4 + 2/4 = 3/4



1/5 + 2/5 = 3/5

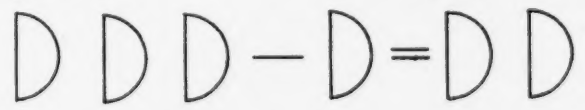
4. Practice with these examples. Demonstrate with concrete examples, pie charts, etc.:

a. 2/2 - 1/2 = 1/2

b. 4/4 - 3/4 = 1/4

c. 8/8 - 5/8 = 3/8

d. 3/4 - 1/4 = 2/4

e. 3/2 - 1/2 = 2/2 

5. Before we can name the sum or difference between two fractional numbers, these fractional numbers must be expressed with the same denominator, which will be the least common multiple of the original denominators.

   Find the "least common denominator" for each set:

   1) $\{1/2, 2/3\}$ 2) $\{7/8, 1/4\}$ 3) $\{1/3, 3/5\}$

   4) $\{1/3, 3/5\}$ 5) $\{1/3, 3/4, 1/2\}$

6. Name the sums and differences in simplest forms. Example:

   $1/2 + 2/6 = \boxed{3}/6 + 2/6 = 5/6$

   $3/4 - 1/2 = 3/4 - \boxed{2}/4 = 1/4$

   a. $2/5 - 1/10 = \square/10 = 3/10$

   b. $3/12 + 2/3 = 3/12 + \square/12 = 11/12$

   c. $3/4 + 2/12 = \square$

   d. $7/8 + 1/4 = \square$

   e. $3/5 + 2/15 = \square$

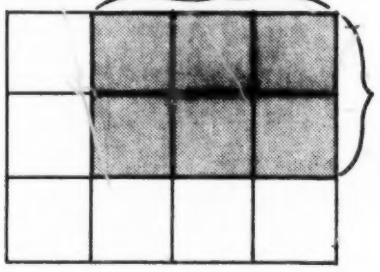
   f. $6/7 - 5/14 = \square/14 = 7/14 = \square$

   g. $5/9 - 1/4 = \square$

OPERATIONS WITH FRACTIONS - MULTIPLICATION

1. Care must be taken not to overemphasize the relationship between successive additions and multiplication involving a whole number factor and fractional factor. If both factors are fractional, then how can the addends be determined? For example: 3/7 x 8/9. To develop understanding of multiplication of rational numbers, three different kinds of physical models can be used. They are (1) rectangular regions, (2) line segments on the number line, and (3) collections of groups of objects.

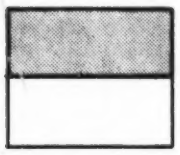
A.



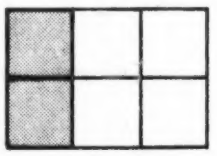
2/3 x 3/4 = 6/12

2. Draw the following on the board:

A



B



C

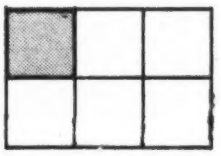
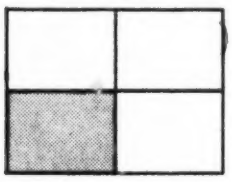


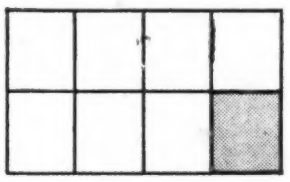
Figure A is a model for what number? (1/2)
Figure B is a model for what number? (1/3)
In Figure C, the crosshatched region is a model for what number? (1/6)
Can we write 1/2 x 1/3 = 1/6? (yes)

3. Allow the pupils to study these pictures and write a mathematical sentence for each:

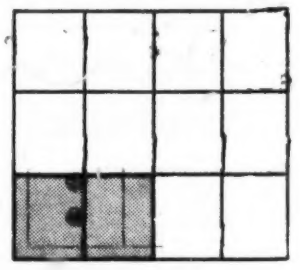
1/2 x 1/2 = 1/4



1/2 x 1/4 = 1/8

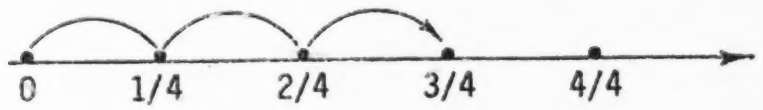


1/3 x 2/4 = 2/12



4. Discuss the following examples with the children:

A.

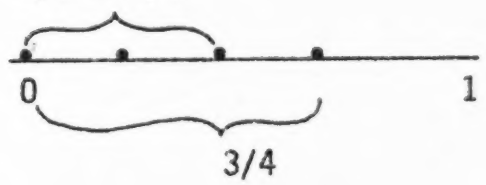


3 jumps of 1/4 3 x 1/4 = 3/4

B.

6 jumps of 1/2 6 x 1/2 = 3

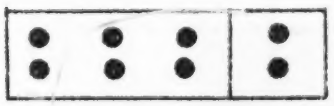
C. 2/3 x 3/4



5. Discuss with your students:

1/4 of the objects in A are colored.
2 x 1/4 equals 2/4 of the objects in B are colored.
3 x 1/4 equals 3/4 of the objects in C are colored.
4 x 1/4 equals 4/4 of the objects in D are colored.

A = (● ○ ○ ○)
B = (● ● ○ ○)
C = (● ● ◑ ○)
D = (● ● ● ●)



3/4 x 8 = 6

2/3 x 3/4 = 1/2

● ● ● ● △

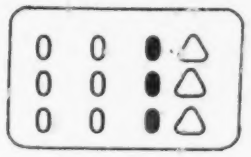
● ● ● ● △

○ ○ ○ ○ △

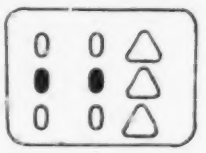
4/5 of the objects are circles.
2/3 of the objects are shaded.
2/3 x 4/5 = 8/15 of the objects are shaded.

3/4 of the rod is shaded.
2/3 of the shaded part is dotted.
2/3 x 3/4 = 6/12 = 1/2

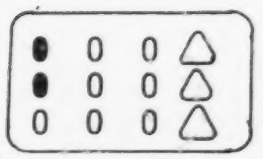
6. Use the sets to help you solve these equations:



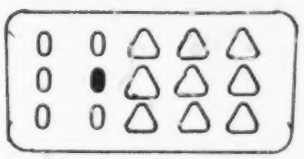
a. 1/3 x 3/4 = a
(a = 1/4)



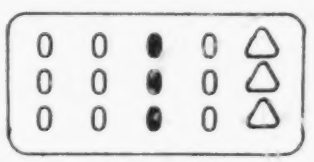
b. 1/3 x 2/3 = b
(b = 2/9)



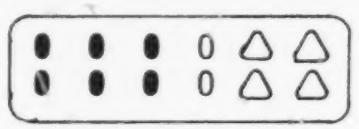
c. 2/9 x 3/4 = c
(c = 1/6)



d. 1/6 x 2/5 = d
(d = 1/15)



e. 1/4 x 4/5 = e
(e = 1/5)



f. 3/4 x 2/3 = f
(f = 1/2)

7. Use the associative principle to find the missing numbers.

a. (5 x 1/2) x 1/3 = 5 x (a x 1/3) = b
a = 1/2 b = 5/6
b. (6 x 1/4) x 1/3 = 6 x (1/4 x e) = f
e = 1/3 f = 6/12
c. (8 x 1/5) x 1/4 = c x (1/5 x 1/4) = d
c = 8 d = 8/20
d. (7 x 1/6) x 1/5 = g x (1/6 x 1/5) = h
g = 7 h = 7/30

8. Use the commutative and associative principles to find the missing numbers.

multiply these first

a. 3/5 x 4/3 = (3 x 1/5) x (4 x 1/3) = 12 x 1/15 = 12/15

then multiply these

b. 2/5 x 3/8 = (2 x 1/5) x (3 x 1/8) = a x b = c
a = 6 b = 1/40 c = 6/40
c. 5/8 x 5/6 = (5 x 1/8) x (5 x 1/6) = d x e = f
d = 25 e = 1/48 f = 25/48
d. 3/4 x 5/6 = (3 x 1/4) x (5 x 1/6) = g x h = i
g = 15 h = 1/24 i = 15/24

VOLUNTEER - MATHEMATICS

OPERATIONS WITH FRACTIONS - DIVISION

1. Review the meaning of division, for example:

$12 \div 3 = 4$ means $12 = 12 = 4 \times 3$

product factors

Tell the pupils that we will think of division of rational numbers as having the same meaning.

$5/8 \div 2/3 = 15/16$ means $5/8 = 2/3 \times 15/16$

product factors

Discuss this example with the pupils: $7/2 \div 3/4 = n$.
What is the multiplication sentence? $7/2 = 3/4 \times n$.
How do we find n? Here is one way. Write $7/2 \div 3/4$ as: $\frac{7/2}{3/4}$

Then $\frac{7/2}{3/4} = \frac{7/2 \times 4/3}{3/4 \times 4/3}$ Why? (Multiply numerator and denominator by 4/3.)

$= 7/2 \times 4/3$ Why? (Reciprocal property)

$= 7/2 \times 4/3$ Why? (Any number divided by 1 is that number)

Then $7/2 \div 3/4 = 7/2 \times 4/3$.

Do several examples, such as these, and allow pupils to <u>discover</u> "invert and multiply."

Study the equations:

$12 \div 3 = 4 \quad 12 \times 1/3 = 4$	Dividing by 3 is the same as multiplying by its reciprocal 1/3.
$8 \div 1/2 = 16 \quad 8 \times 2 = 16$	Dividing by 1/2 is the same as multiplying by its reciprocal 2.

2. These exercises are a review of the idea of a reciprocal:

A. $r = 3/4 \times 4/3$ $\quad r = 5/2 \times 2/5$
$r = 7/8 \times 8/7$ $\quad 25/3 \times 3/25 = r$

B. What is the product for each part of exercise A? (1)

C. What do you notice about the factors in exercise A? (numerator and denominator are interchanged)

D. Find n in each of the following:

$2/3 \times n = 1$ (3/2)	$n \times 12/5 = 1$ (5/12)
$n \times 8/10 = 1$ (10/8)	$n \times 1 = 1$ (1)
$7/5 \times n = 1$ (5/7)	$0 \times n = 1$ (no answer)

E. Is one its own reciprocal? (yes)

F. Does zero have a reciprocal? (no)

3. Use reciprocals to solve the equations:

1. $8 \div 4 = 8 \times a = b$
2. $9 \div 3 = 9 \times c = d$
3. $10 \div 5 = 10 \times e = f$
4. $12 \div 6 = 12 \times g = h$
5. $18 \div 9 = 18 \times i = j$
6. $21 \div 7 = 21 \times k = L$
7. $24 \div 8 = 24 \times m = n$
8. $7 \div 1/2 = 7 \times p = q$
9. $9 \div 1/5 = 9 \times r = s$
10. $6 \div 1/9 = 6 \times t = u$

(a = 1/4, b = 2, c = 1/3, d = 3, e = 1/5, f = 2, g = 1/6, h = 2, i = 1/9, j = 2, k = 1/7, L = 3, m = 1/8, n = 3, p = 2, q = 14, r = 5, s = 45, t = 9, u = 54)

4. Use reciprocals to name the quotient. For example:

$11 \div 1/2 = 11 \times 2 = 22$ (The reciprocal for 1/2 is 2)

1. $16 \div 1/3$ $(16 \times 3 = 48)$
2. $7 \div 1/9$ $(7 \times 9 = 63)$
3. $6 \div 3/5$ $(6 \times 5/3 = 30/3 = 10)$
4. $9 \div 4/3$ $(9 \times 3/4 = 27/4 = 6\ 3/4)$
5. $15 \div 3/5$ $(15 \times 5/3 = 75/3 = 25)$
6. $48 \div 6/7$ $(48 \times 7/6 = 336/6 = 56)$

5. Circle the word that makes the sentence true.

a. When a whole number is multiplied by 1/2, the answer is (greater, (less)) than when the whole number is multiplied by 2.

b. When a whole number is divided by 1/2, the answer is ((greater), less) than when the whole number is divided by 2.

6. Write the correct symbol. Do as little computing as possible.

a. $8964 \times 1/2\ (<)8964 \times 2$
b. $686 \div 1/2(>)686 \div 2$
c. $945 \div 1/3(>)945 \div 3$
d. $3/4 \times 4/5(<)3/4$
e. $5/6 \times 2/7(<)5/6$
f. $18/20 \div 2/3(>)18/20 \div 3/2$
g. $5/6 \div 6/5(<)6/5 \div 5/6$

BIBLIOGRAPHY

If the following are available in the building where you are serving, we recommend their use.

Aftermath (with Duplicating Masters), Creative Publications, P.O. Box 328, Palo Alto, 94302

The Arithmetic Teacher, National Council of Teachers of Math

A Cloudburst of Mathematics Lab Experiments, Midwest Publications, Birmingham, Michigan

Computational Skills Development Kit, SRA, Chicago

Early Childhood Education in Oklahoma, State Department of Education, 1971

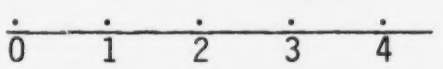
Experiences in Mathematical Ideas, NCTM, Washington

Experiments in Mathematics, Houghton Mifflin Co., Boston

The Franklin Mathematics Series, Lyons and Carnahan, Chicago

GLOSSARY

abacus	a device used for calculating, usually involving sliding beads or counters along a wire
addend	any one of a set of numbers to be added
binary operation	an operation in which two elements are required, as in addition, subtraction, multiplication, division - example: in adding 4 + 7 + 6 + 3, only two of these elements can be added together at any one time
closure	within any set you will always have a member of that set in your result - example: 3 + 2 = 5 (both addends and sum are whole numbers). Closure always results in addition and multiplication but not always in subtraction and division - example: 5 - 7 = n (n is not a whole number)
composite number	a whole number greater than 1, that has more than 2 factors - example: 6-factors 1, 2, 3, 6
coordinate	a symbol assigned to a point to which it can be referred
counting number	any positive number used for counting - example: 1, 2, 3, etc.
difference	the number resulting from the subtraction operation
distributive (multiplication-addition) principle	breaking apart a number before multiplying - example: 6 x 24 = (6 x 20) + (6 x 4) or a (b + c) = a x b + a x c
dividend	example: in the problem 33 ÷ 7, 33 is the dividend
divisor	example: in the problem 33 ÷ 7, 7 is the divisor
element	a member of a set
equation	a mathematical sentence involving the use of the equality symbol - example: 5 + 4 = 9
factor	example: in the equation 6 x 7 = 42, both 7 and 6 are factors of 42

grouping (associative) principle	when adding or multiplying numbers, the grouping can be changed but the sum or product will remain the same - example: (8 + 2) + 6 = 8 + (2 + 6)
identity (additive)	that number which does not change the number to which it is added - example: 0 + 3 = 3
identity (multiplicative)	that number which does not change the number it is multiplied - example: 1 x 5 = 5
integers	the whole numbers, together with their corresponding negatives - example: 0, 1, 2, 3, etc., and -1, -2, -3, etc.
inverse principle	examples: a + b = c; therefore, c - a = b or c - b = a, and 3 x 4 = 12; therefore, 12 ÷ 4 = 3 and 12 ÷ 3 = 4
multiple	example: in the problem 4 x 6 = 24, 24 is a multiple of both 6 and 4, as it is also of 12 and 2, 8 and 3, and 24 and 1
number line	a line on which specified points are given number labels or names - example: 
numerator	the number above the line in a fraction symbol - example: 2/3, 2 is the numerator
operation	a specific process for combining or separating quantities - examples: addition, subtraction, multiplication, division
order (commutative) principle	when adding or multiplying two numbers, the order of the addends or factors does not affect the sum or product - examples: 4 + 5 = 5 + 4, or 3 x 5 = 5 x 3
place value	the value given to any digit in a numeral based upon its position in that numeral (the number base usually is 10) - example: in the numeral 3,257, the 5 stands for 50 or 5 tens; the 2 for 200 or 2 one hundreds, and the 3 for 3,000 or 3 one thousands
prime number	a number greater than 1 whose only factors are itself and 1 - example: 7 (this cannot be divided evenly by any number smaller than itself or greater than 1)
product	the result of the multiplication operation - example: in 6 x 7 = 42, 42 is the product of 6 and 7

quotient	the number (other than the remainder) that is the result of the division operation - example: $7\overline{)47}$ — 6 - quotient; 42; 5 - remainder
real number	any positive or negative number, plus zero and including decimals, which can be written as a fraction - example: 0, 1, 2, 3, etc.; -1, -2, -3, etc.; 3/4, 1/2, etc.; 5.6, 3.4, etc.
reciprocal	two numbers are reciprocals of each other if their product is 1 - example: $4/7 \times 7/4 = 1$
set	a group or collection of objects, numbers, etc.
whole number	any counting number plus 0. Example: 0, 1, 2, 3, etc.